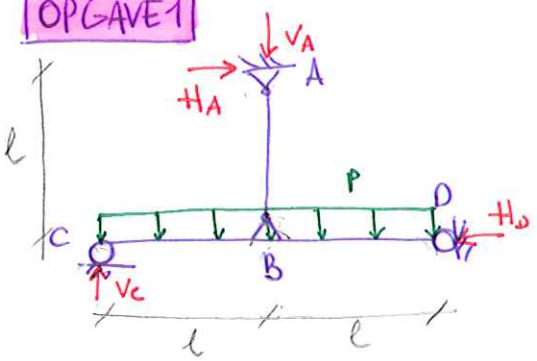
REEKS 0

OPGAVE 1



Isostatisch?

$ls,tot = ni - ne = (3b + l - r) - (3n - m)$

(# reacties, # knopen, # onb, # vgln, # staven, # reductie-eln, # triv. vgln (interne enkelv. mech.))

$ls,tot = (3.3 + 4 - 2) - (3.4 - 1) = 0$

of korter: staaf tussen 2 nh! $M=0$ $D=0$ i.v.m. scharnier.

$ls,tot = 3.t - \#DOF = 3.2 - (2 + 2 + 1 + 1) = 0$

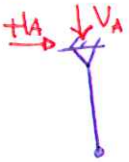
(# mazen (gesloten!))

HE: $H_A = H_D$

VE: $V_A - V_C = -2pl$

ME(A): $V_C . L + H_D . L = 0 \rightarrow V_C = -H_D$

extra: $M_B = 0$:

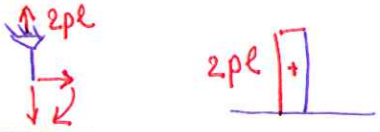


$H_A . l = 0$

schanier is tussen AB, dus CD blijft 1 geheel

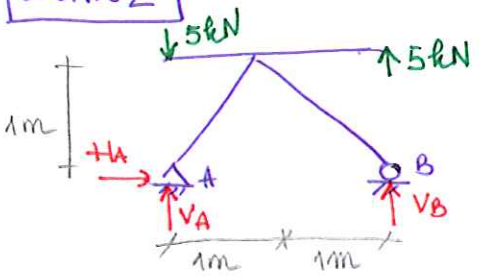
$H_A = 0$
$H_D = 0$
$V_C = 0$
$V_A = -2pl$

dus een trekkracht in staaf AB



AB is een "pendel"
→ enkel normaalkrachten.

OPGAVE 2



$ls,tot = (3b + l - r) - (3n - m)$
$= (3.1 + 3 - 0) - (3.2 - 0) = 0$

$ls,tot = 3t - \#DOF = 3.1 - (2 + 1) = 0$

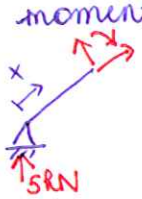
HE: $H_A = 0$

VE: $V_A = -V_B$

ME(A): $V_B . 2m + 5kN . 2m = 0$ ⇒

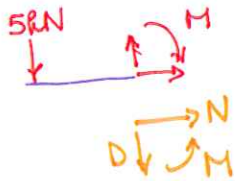
$V_B = -5kN$
$V_A = 5kN$

momentendiagramma.



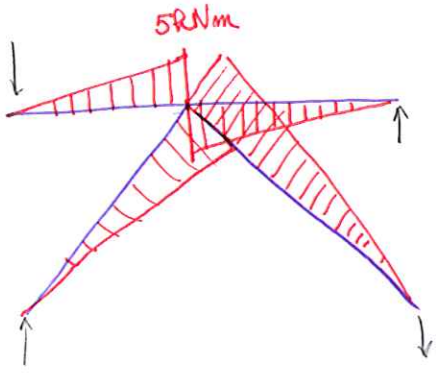
$M = -5 . \frac{\sqrt{2}}{2} . x$

$x = 0 \quad M = 0$
$x = \sqrt{2} \quad M = -5kNm$

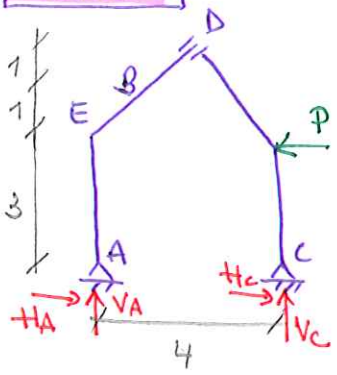


$M = -5kN . x$
$x = 0 \quad M = 0$
$x = 1 \quad M = -5kNm$

teken moment aan kant getrokken vezel.



OPGAVE 3



$I_{n,tot} = 3t - \#DOF = 3.1 - (1+1+1) = 0$

kracht P wordt doorgegeven naar punt D door de staaf. De kracht wordt niet doorgegeven naar E door de inwendige glijder
→ normaalkracht in B = 0

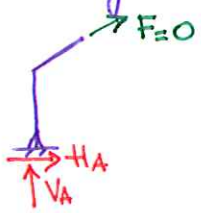
HE: $H_A + H_C = P$

VE: $V_A + V_C = 0$

ME(A): $V_C \cdot 4m + P \cdot 3m = 0 \Rightarrow$

$$\boxed{\begin{aligned} V_C &= -\frac{3P}{4} \\ V_A &= +\frac{3P}{4} \end{aligned}}$$

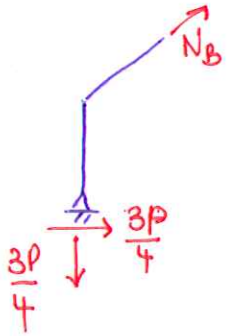
Extra vgl: scharnier in punt D ⇒ in de richting van staaf DE kan geen kracht worden doorgegeven → snede krachtenevenwicht in richting staaf DE:



$$\frac{H_A}{\cos 45°} + \frac{V_A}{\sin 45°} = 0 \Rightarrow H_A = -V_A$$

$$\boxed{\begin{aligned} H_A &= -\frac{3P}{4} \\ H_C &= \frac{7P}{4} \end{aligned}}$$

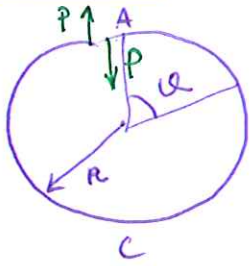
Snede in B:



$$N_B = -\left(\frac{-\frac{3}{4}P}{\cos 45} + \frac{\frac{3}{4}P}{\sin 45}\right)$$

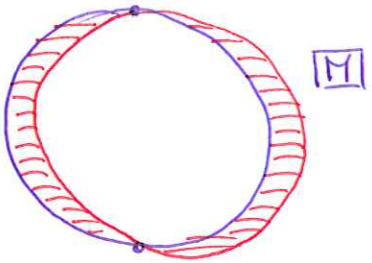
$\Rightarrow \boxed{N_B = 0}$ logisch want glyder ...

OPGAVE 4

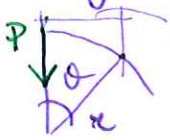


A en C op lijn van de krachten

gemakkelijk te zien dat:



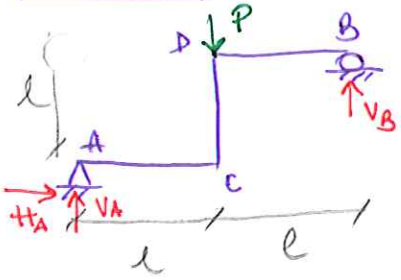
Analytisch:



moment = kracht × hefboomarm

moment : $M = P \cdot x \sin\alpha$

OPGAVE 5



$I_{s,tot} = 3.t - \#DOF = 3.1 - (1+2) = 0$

HE: $H_A = 0$

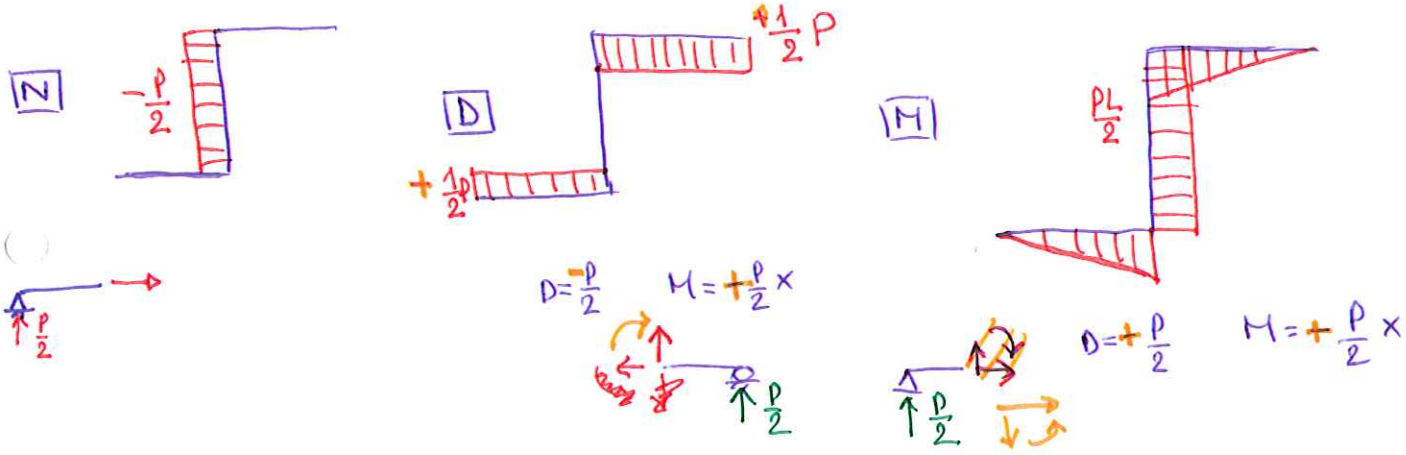
VE: $V_A + V_B = P$

ME(A): $V_B . 2l = P . l \Rightarrow$

$V_B = \frac{P}{2}$

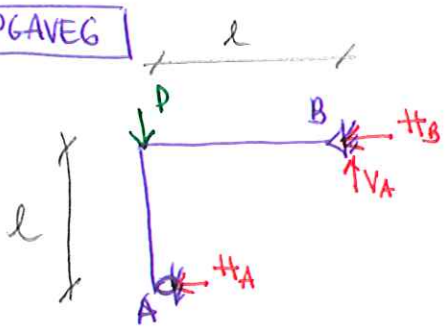
$V_A = \frac{P}{2}$

Reductiediagramma:



Uit de reductediagramma is duidelijk af te lezen dat staaf CD bepalend zal zijn voor de dimensionering.

OPGAVE 6



$I_{s,tot} = 3.1 - (2+1) = 0$

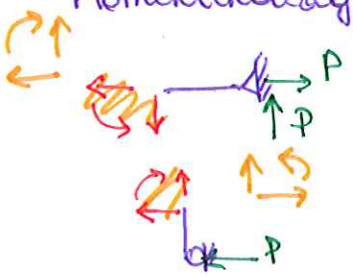
HE : $H_A + H_B = 0$

VE : $V_A = P$

ME(B) : $H_A . l = P . l$

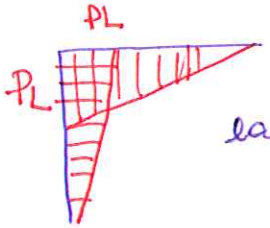
$$\boxed{\begin{aligned} H_A &= P \\ H_B &= -P \\ V_A &= P \end{aligned}}$$

Momentendiagram



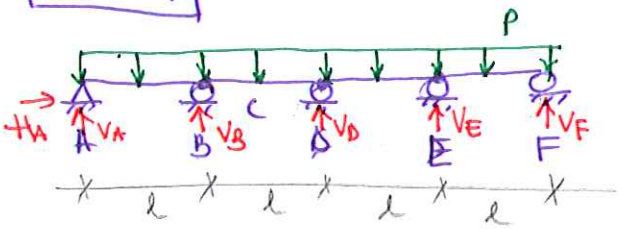
$M = P . x$

$M = P . x$



langs kant getrokken wel !

OPGAVE 7



$I_{s,tot} = 3t - \#DOF$

$= 3.4 - (1+2+2+2+2+1+1+1) = 0$

HE : $H_A = 0$

VE : $V_A + V_B + V_D + V_E + V_F = 4P . l$

ME(A) : $V_B . l + 2V_D l + 3V_E l + 4V_F l = 4Pl . 2l$

$\Leftrightarrow V_B + 2V_D + 3V_E + 4V_F = 8Pl$

Extra vgln doorrij inwendige scharnieren.

ME(B) : $V_A . l = P . l \frac{l}{2} \Rightarrow \boxed{V_A = \frac{Pl}{2}}$

ME(D) : $V_A . 2l + V_B . l = 2Pl^2$

$2V_A + V_B = 2Pl$

$V_B = 2Pl - Pl \Rightarrow \boxed{V_B = Pl}$

ME(E) : $3V_A l + 2V_B l + V_D l = 3Pl \frac{3}{2} l$

$3V_A + 2V_B + V_D = \frac{9}{2} Pl$

$V_D = \frac{9}{2} Pl - \frac{3Pl}{2} - 2Pl \Rightarrow \boxed{V_D = Pl}$

andere kant :

$V_F . l = \frac{Pl^2}{2} \Rightarrow \boxed{V_F = \frac{Pl}{2}}$

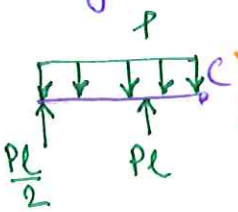
ME(A) :

$\Rightarrow V_B + 2V_D + 3V_E + 4V_F = 8Pl \Leftrightarrow Pl + 2Pl + 3V_E + 2Pl = 8Pl$

$\Leftrightarrow V_E = \frac{8Pl - Pl - 2Pl - 2Pl}{3}$

$$\boxed{V_E = Pl}$$

Buigmoment in C

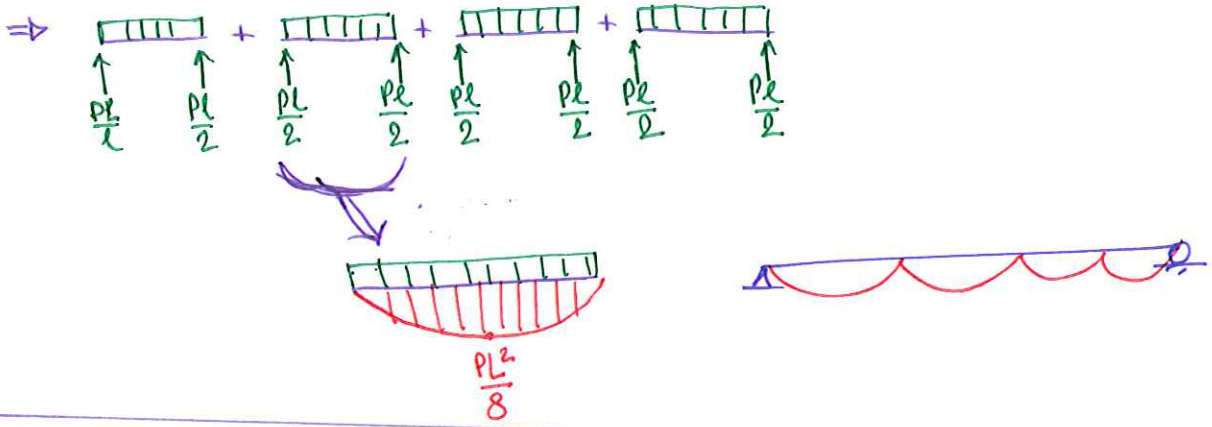


$$M = -\frac{Px^2}{2} + \frac{Pl}{2}x + Pl(x-l)$$

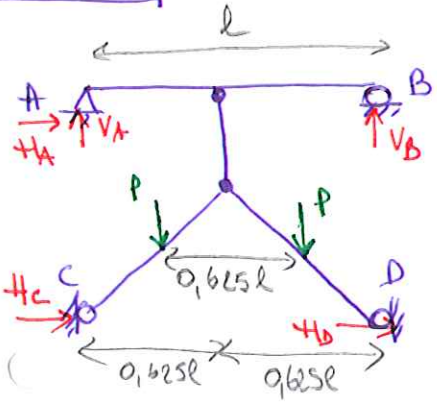
in C : $x = \frac{3}{2}l$

$$M_c = \frac{P}{2}\frac{9}{4}l^2 - \frac{P}{2}l\frac{3}{2}l - Pl\left(\frac{3}{2}l - l\right)$$

$$\boxed{M_c = -\frac{1}{8}Pl^2}$$

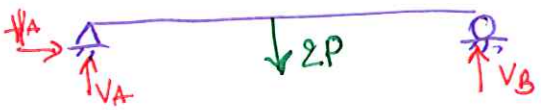


OPGAVE 8



de staaf met 2 scharnieren is een "pendel" want er zullen alleen normaalkrachten opgenomen worden.

vereenvoudigen van de structuur :

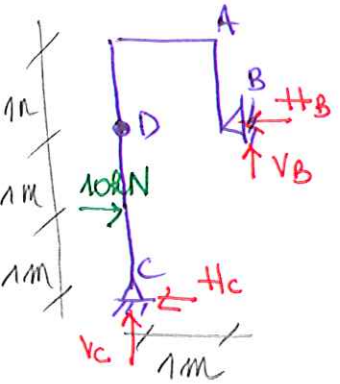


momentendiagram.

puntlast $M_{max} = \frac{F.L}{4}$ $\quad F = 2P$

dus $M_{max} = \frac{Pl}{2}$

OPGAVE 9



$I_{stat} = 3.1 - (1+1+1) = 0$

HE: $H_B + H_C = 10kN$

VE: $V_C + V_B = 0$

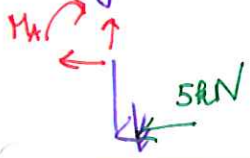
ME(A): $-10kN.1m + H_B.2m + V_B.1m = 0$

$V_B + 2H_B = 10kN$

ME(C): extra vgl: $10kN.1m = H_C.2m$

$H_C = 5kN$
$H_B = 5kN$
$V_B = 0$
$V_C = 0$

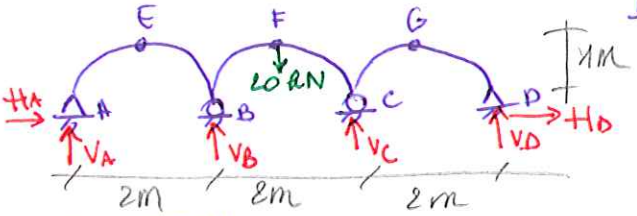
Buigmoment in A:



$M_A = -5kN.x$

A: $x = 1m$ $\quad M_A = -5kNm$

OPGAVE 10



$I_{stat} = 3.3 - (1+2+2+1+1+1+1) = 0$

HE: ~~$H_A + H_B$~~ $H_A + H_D = 0$

VE: $V_A + V_B + V_C + V_D = 20kN$

$V_B = V_C$ $\quad V_A = V_D$

symmetrische structuur + symm last

$2V_A + 2V_B = 20kN$

$V_A + V_B = 10kN$

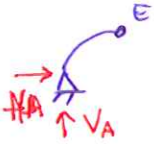
ME(A): $V_B.2m + V_C.4m + V_D.6m = 20kN.3m$

$V_B = V_C$ door symm.

$V_B.6m + V_D.6m = 20.3m \Rightarrow V_B + V_D = 10kN$

logisch want $V_D = V_A$.

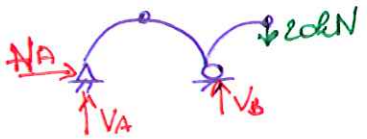
extra ME(E)



$1m.V_A - 1m.H_A = 0$

$V_A = H_A$

extra ME(F)



$3m.V_A + 1m.V_D - 1m.H_A = 0$

$3V_A + V_B = H_A$

$V_A + V_B = 10kN$

$H_A + H_D = 0$

$V_A = H_A$

$3V_A + V_B = V_A \rightarrow 2V_A + V_B = 0$

$V_B = -2V_A \rightarrow V_A + V_B = 10kN$

$V_A - 2V_A = 10kN$

$V_A = -10kN$

$V_A = -10kN = V_D$
$V_B = 20kN = V_C$
$H_A = -10kN$
$H_D = 10kN$

Buigmomenten in B en C?
→ hier niet noodzakelijk nul
normaal wel in scharnierpunt:

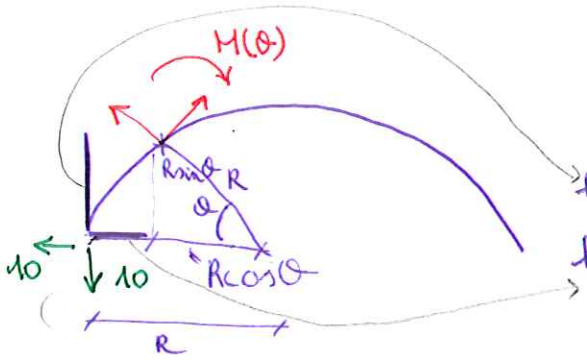
maar

is equivalent aan

starre verbinding

↓

geeft mogelijk wel M door van een op ander staaf.



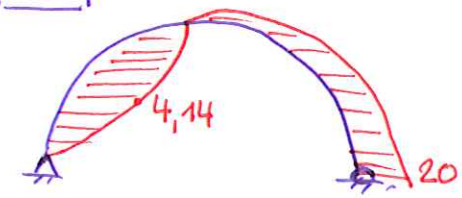
hefboomarm horizontale kracht = $R\sin\theta$
hefboomarm verticale kracht = $R - R\cos\theta$

$$M(\theta) = -10R\sin\theta + 10(R - R\cos\theta)$$

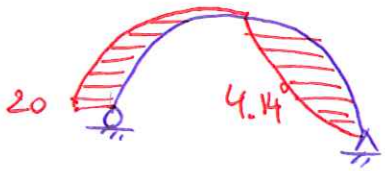
$\theta = 135°$ $M = 1$
$\theta = 45$ $M = -4{,}14$

$\theta = 90°$ $M = 0$
$\theta = 0°$ $M = 0$
$\theta = 180°$ $M = 20$

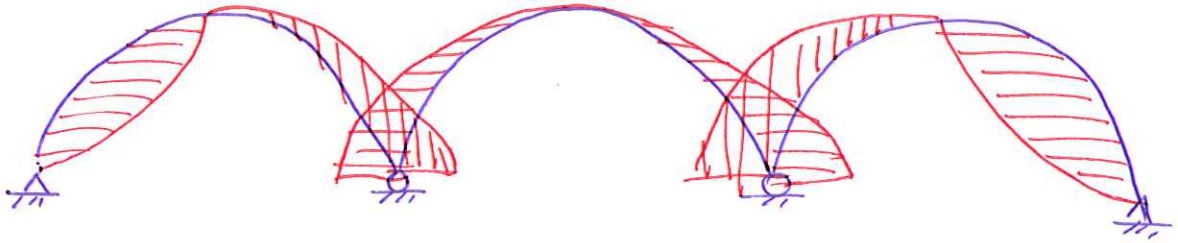
$|M_B| = 20$



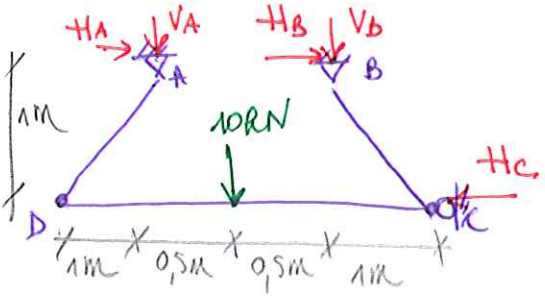
$|M_C| = 20$ → gespiegeld want symm. structuur



Totaal momentenverloop:



OPGAVE 11



$I_{s,tot} = 3.2 - (1+1+2+1+1) = 0$

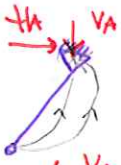
HE : $H_A + H_B = H_C$

VE : $V_A + V_B = -10kN$

M(A) : $V_B \cdot 1m + 10kN \cdot 0{,}5m + H_C \cdot 1m = 0$

$\Leftrightarrow \quad V_B + H_C = -5kN$

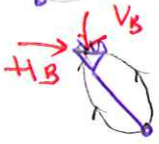
Extra $M_D = 0$:



$V_A \cdot 1m + H_A \cdot 1m = 0$

$V_A + H_A = 0$

Extra $M_C = 0$



$V_B \cdot 1m - H_B \cdot 1m = 0$

$V_B - H_B = 0$

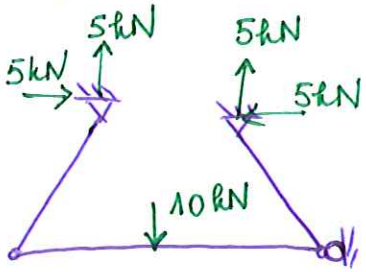
$$\begin{cases} H_A + H_B = H_C \\ V_A + V_B = -10kN \\ V_B + H_C = -5kN \\ V_A + H_A = 0 \\ V_B - H_B = 0 \end{cases}$$

$$\begin{cases} V_B = H_B \\ V_A = -H_A \\ V_A = -10kN - V_B \\ V_A + H_B = H_C \\ H_C = -5kN - H_B \end{cases}$$

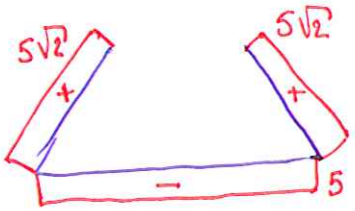
$$\begin{cases} -10kN + V_B + V_B = -5kN - V_B \\ 3V_B = -15kN \\ V_B = -5kN \end{cases}$$

$\Rightarrow$

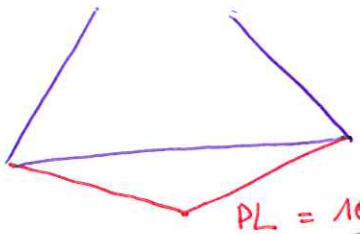
$V_B = -5kN$
$V_A = -5kN$
$H_C = 0$
$H_A = 5kN$
$H_B = -5kN$



N



M

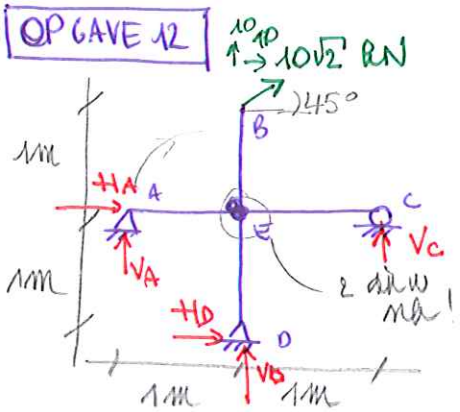


$\frac{PL}{4} = \frac{10kN \cdot 3m}{4} = 7{,}5kNm$

↓

staven waarvan de 2 uiteinden scharnier zijn, kunnen geen M hebben als ze enkel in de knopen belast zijn.

OPGAVE 12



$l_{b,tot} = 2.3 - (1+1+2+1+1) = 0$

HE: $H_A + H_D + 10\,kN = 0$

VE: $V_A + V_C + V_D + 10\,kN = 0$

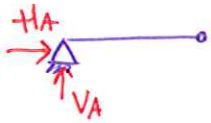
ME(A): $V_D \cdot 1m + H_D \cdot 1m + V_C \cdot 2m = 0$

↳ $10\sqrt{2}\,kN$ zit hier niet mee in

↳ de drager ervan gaat door A!

$\Rightarrow V_D + H_D + 2V_C = 0$

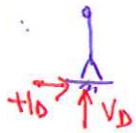
extra E:



ME(E): $V_A \cdot 1m = 0$

$V_A = 0$

extra E:



ME(D): $H_D \cdot 1m = 0$

$H_D = 0$

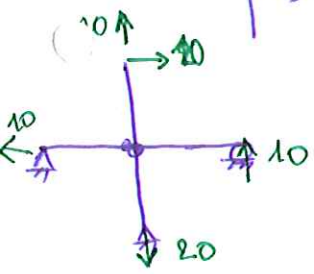
$\Rightarrow H_A = -10\,kN$

$\Rightarrow$

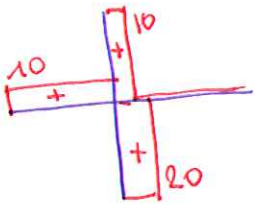
$$\begin{cases} H_A = -10kN \\ V_A = 0 \\ H_D = 0 \\ V_C + V_D = -10kN \\ V_D + 2V_C = 0 \end{cases}$$

$$\begin{cases} V_D = -2V_C \\ V_C - 2V_C = 10kN \\ V_C = +10kN \\ V_D = -20kN \end{cases}$$

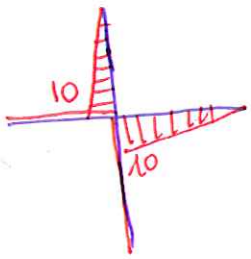
$$\boxed{\begin{array}{l} V_A = 0 \\ H_A = 0 \\ V_C = +10kN \\ V_D = -20kN \\ H_D = 0 \end{array}}$$

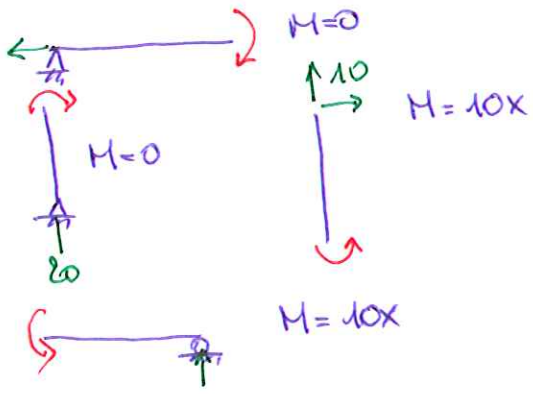


N

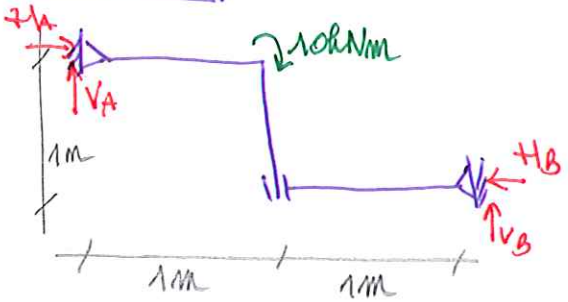


M





OPGAVE 13



$I_{s,tot} = 3.1 - (1+1+1)$
⇑ ⇑ ⫴

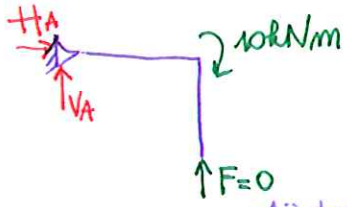
HE: $H_A = H_B$

VE: $V_A + V_B = 0$

ME(A): $10kNm + H_B \cdot 1m - V_B \cdot 2m = 0$

$H_B - 2V_B = -10kN$

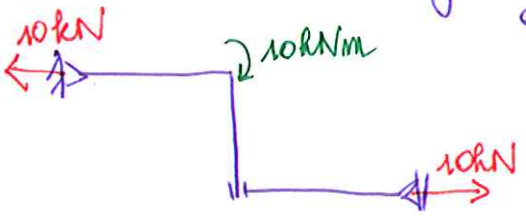
extra



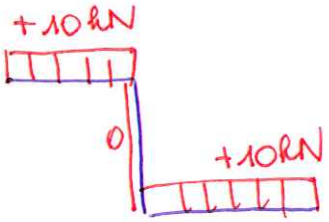
$V_A = 0$ ⇒

$V_A = 0$
$V_B = 0$
$H_B = -10kN$
$H_A = -10kN$

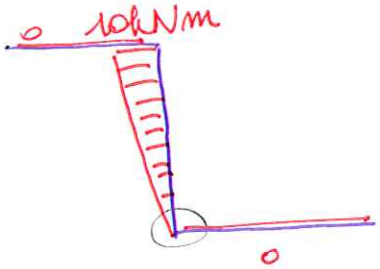
glijder geeft geen normaalkr door.



N

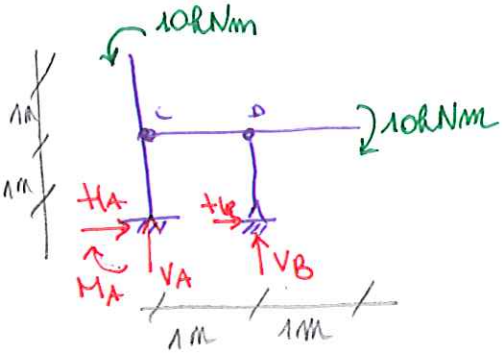


M



naar glijder want 0.

OPGAVE 14



$I_{s,tot} = 3.1 - (1-1-1) = 0$
↔ ↕

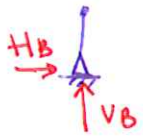
HE : $H_A + H_B = 0$

VE : $V_A + V_B = 0$

ME(A) : $M_A + 10kNm - 10kNm - V_B . 1m = 0$

$M_A = V_B . 1m$

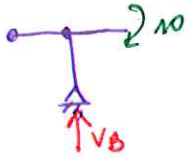
extra $M_D = 0$



$H_B . 1m = 0$

$H_B = 0$

extra $M_C = 0$



$10kNm = V_B . 1m$

$\Rightarrow V_B = 10kN$

$\Rightarrow$

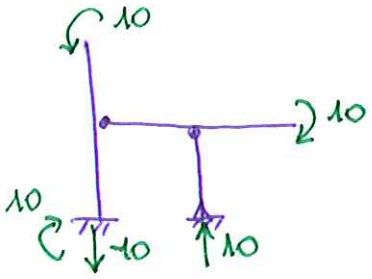
$V_B = 10kN$

$V_A = -10kN$

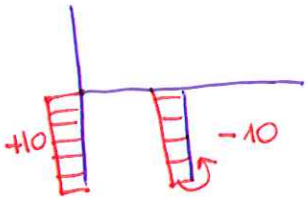
$H_B = 0$

$H_A = 0$

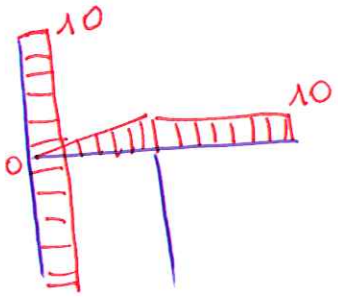
$M_A = 10kNm$



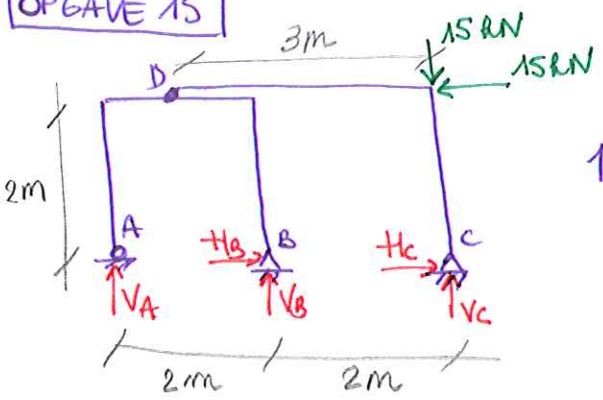
N



M



OPGAVE 15

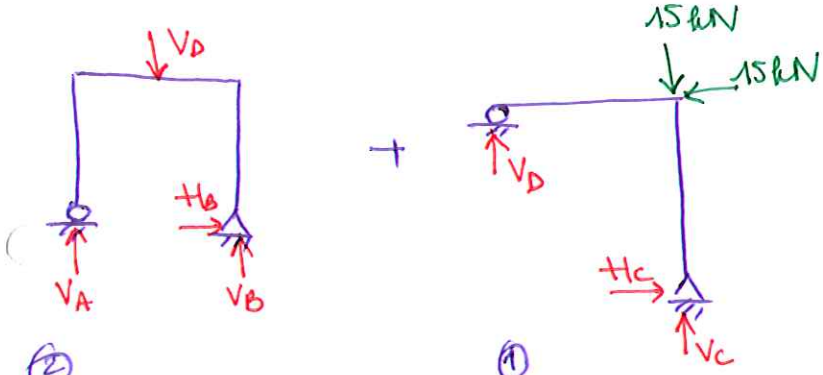


punt D = rod!

$I_{s,tot} = 3.2 - (2 + 2.1 + 2)$

$= 6 - 6 = 0$

best opsplitsen in 2:



(2)

HE: $H_B = 0$

VE: $V_A + V_B = 10kN\ (= V_D)$

ME(A): $\cancel{0}.1m = V_B . 2m$

$\boxed{V_B = 5\,kN}$

$\boxed{V_A = 5kN}$

(1)

HE: $\boxed{H_C = 15kN}$

VE: $V_D + V_C = 15kN$

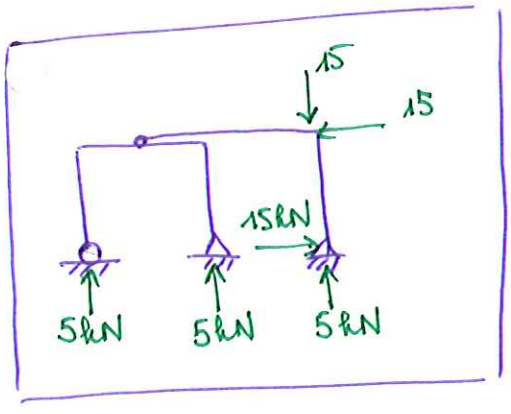
ME(D): $V_C . 3m + H_C . 2m = 15kN . 3m$

$3V_C + 2H_C = 45kN$

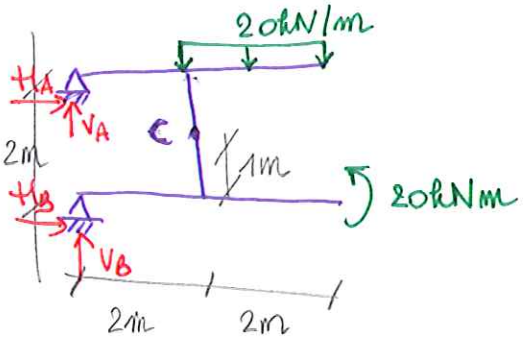
$3V_C = 45kN - 2 . 15kN$

$\boxed{V_C = 5\,kN}$

$\boxed{V_D = 10kN}$



OPGAVE 16



$l_{o,tot} = 1.3 - (1+1+1) = 0$

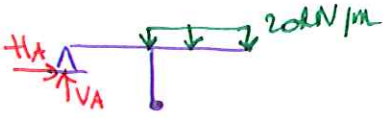
HE: $H_A + H_B = 0$

VE: $V_A + V_B = 20\frac{kN}{m} \cdot 2m = 40kN$

ME(A): $H_B \cdot 2m + 20kNm = 20\frac{kN}{m} \cdot 2m \cdot 3m$

$\rightarrow$ $H_B = 50kN$, $H_A = -50kN$

extra $M_C = 0$

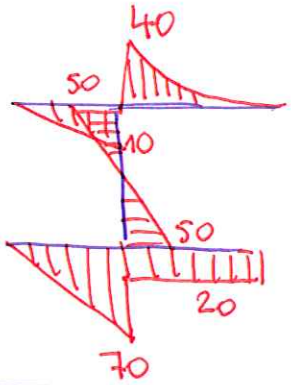


$V_A \cdot 2m + H_A \cdot 1m + 20\frac{kN}{m} \cdot 2m \cdot 1m = 0$

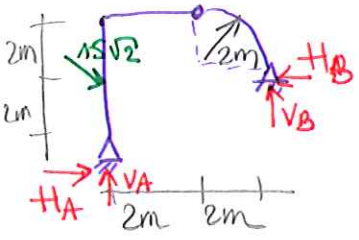
$2V_A + H_A = -4kN$

$\rightarrow$ $V_A = 5kN$, $V_B = 35kN$

M



OPGAVE 17



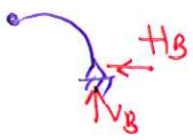
HE: $H_B - H_A = 15kN$

VE: $V_A + V_B = 15kN$

ME(A): $15m \cdot 2m = H_A \cdot 4m + V_B \cdot 4m$

$\rightarrow$ $2V_B + H_A = 15kN$

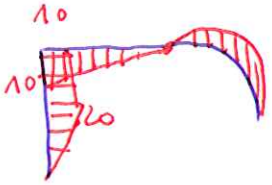
extra $M_C = 0$



$H_A \cdot 2m = V_B \cdot 2m$

$\Rightarrow$ $H_A = V_A = 5kN$, $V_B = 10kN$, $H_B = -10kN$

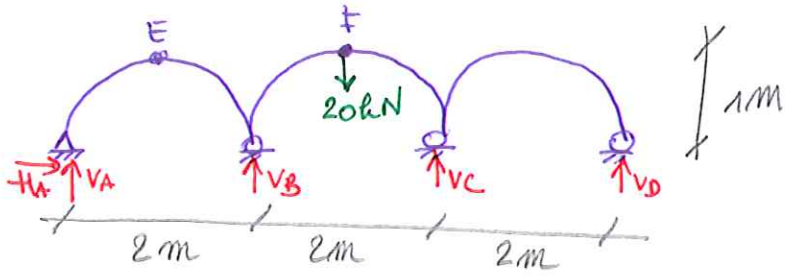
M



$M(\theta) = 5(2 - 2\cos\theta) - 5 \cdot 2\sin\theta$

$= 10(1 - \cos\theta - \sin\theta)$

OPGAVE 18



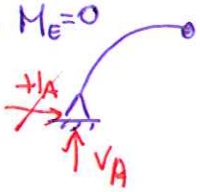
HE: $H_A = 0$

VE: $V_A + V_B + V_C + V_D = 20kN$

ME(A): $V_B \cdot 2m + V_C \cdot 4m + V_D \cdot 6m = 20kN \cdot 3m$

$\rightarrow 2V_B + 4V_C + 6V_D = 60kN$

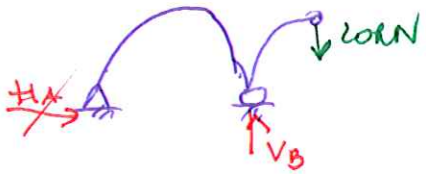
uitna: $M_E = 0$



$V_A \cdot 1m = 0$

$V_A = 0$

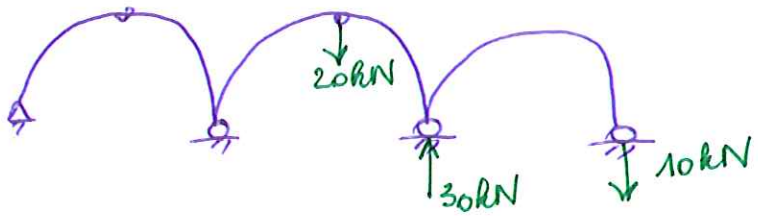
uitna $M_F = 0$



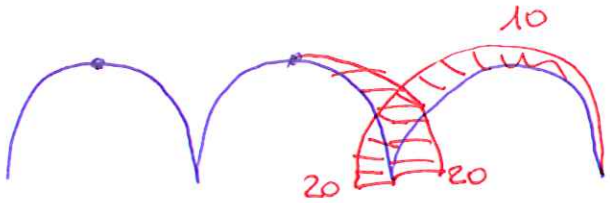
$V_B \cdot 1m = 0$

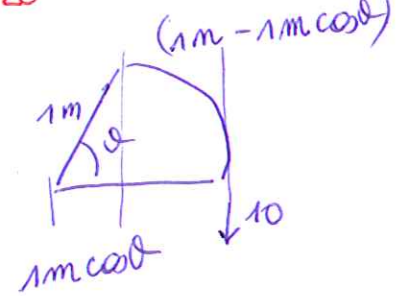
$V_B = 0$

$$\Rightarrow \begin{cases} V_C + V_D = 20kN \\ \cancel{4V_C + 6V_D = 60kN} \\ 2V_C + 3V_D = 30kN \end{cases} \Rightarrow \boxed{\begin{array}{l} V_C = 30kN \\ V_D = -10kN \end{array}}$$



M





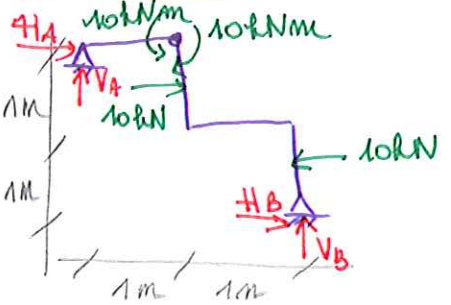
$M(\theta) = 10(1 - \cos\theta)$

$\theta = 0°$ $M(\theta) = 0$

$\theta = 90°$ $M(\theta) = 10kNm$

$\theta = 180°$ $M(\theta) = 20kNm$.

OPGAVE 19



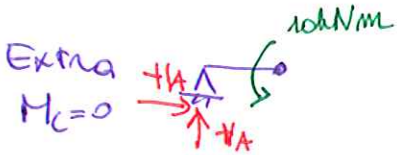
HE: $H_A + H_B = 10kN - 10kN = 0$

VE: $V_A + V_B = 0$

ME(A): $10kNm - 10kNm + 10kN \cdot 0{,}5m - 10kNm \cdot 1{,}5m + H_B \cdot 2m + V_B \cdot 2m = 0$ (5kN, 15kN)

$\Rightarrow H_B + V_B = 5kN$

Extra $M_C = 0$

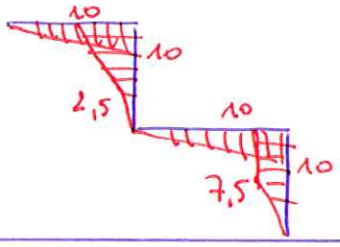


$10kNm - V_A \cdot 1m = 0$

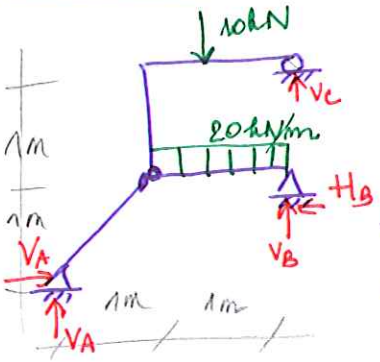
$\rightarrow$

$V_A = 10kN$	$H_B = 15kN$
$V_B = -10kN$	$H_A = -15kN$

M



OPGAVE 20



$I_{s,tot} = 3 \cdot 2 - (2 + 1 + 1 + 1 + 1)$

$\rightarrow$ intern scharnier in D verbindt 3 $\neq$ delen wtd constructie $\rightarrow$ 2 vgln tgv D uitschrijven.

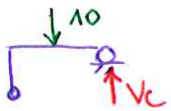
HE: $H_A = H_B$

VE: $V_A + V_B + V_C = 10kN + 20\frac{kN}{m} \cdot 1m = 30kN$

ME(A): $2m \cdot V_B + 2m \cdot V_C + 1m \cdot H_B = 10kN \cdot 1{,}5m + 20\frac{kN}{m} \cdot 1m \cdot 1{,}5m$

$\Rightarrow 2V_B + 2V_C + H_B = 45kN$

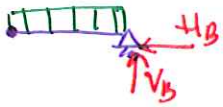
Extra:



$V_C \cdot 1m = 10kN \cdot 0{,}5m$

$\boxed{V_C = 5kN}$

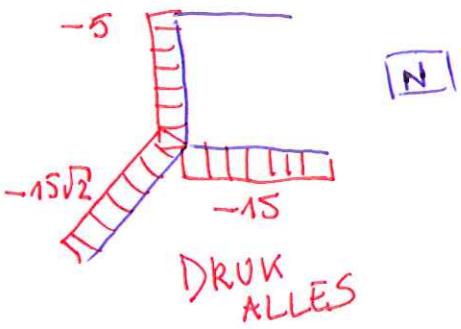
Extra



$V_B \cdot 1m = 20kN/m \cdot 1m \cdot 0{,}5m$ $\quad \boxed{V_B = 10kN}$

$\Rightarrow$ $\boxed{H_B = 15kN}$ $\quad \boxed{H_A = 15kN}$ $\quad \boxed{V_A = 15kN}$

N



M