On the solution of the EPR paradox and the explanation of the violation of Bell's inequality

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Abstract

A new theory is proposed offering a consistent conceptual basis for nonrelativistic quantum mechanics. The Einstein-Podolsky-Rosen (EPR) paradox is solved and the violation of Bell's inequality is explained by maintaining realism, inductive inference and Einstein separability.

1 Introduction

Quantum theory is obviously the greatest achievement in theoretical physics of the 20th century. It has been successfully applied to the description of electrons in atoms, molecules and solids, of nuclei, radiation, elementary particles - an enormous number of different phenomena. Nevertheless, it is astonishing that during all these successes the basic concepts, especially the measurement and the collapse of the wave function [1] (when we confine our considerations to nonrelativistic quantum mechanics only) has remained unclear and controversial, essentially no progress has been achieved for almost 70 years, only the confusion has increased. This unprecedental situation has lead to an irrational 'folklore' in the physics community: most physicists settle for themselves the deep and tantalizing fundamental problems of quantum mechanics by telling that all this is philosophy (and this is not meant to be a compliment), and for all practical purposes standard quantum mechanics is completely satisfactory. Indeed, the last statement is true as long as one compares experimental cross-sections, transition probabilities or spectral wave numbers with the theoretical predictions. Somehow this part of the theory is not sensitive to the logical consistency of the basic concepts.[1] Nevertheless, the fundamental problems of quantum mechanics cannot be considered as mere philosophical questions, as two famous results, the EPR paradox[2] and the violation of Bell's inequality[3] clearly demonstrate. Both of these seem to imply that quantum mechanics violates Einstein separability[2], an obvious physical requirement which follows from the principle of locality and means that separated systems (i.e., which are prevented from any interaction with each other) cannot influence each other. When a physical theory violates a well established, basic physical principle, that is certainly not a philosophical problem.

In order to look at the problem more closely let us briefly recall the essence of the two results mentioned above. The EPR paradox draws the fact to our attention that if we are given two separated systems in an entangled state (which is the result of a previous interaction), and we perform a measurement on one of the systems, then, according to the usual rules of quantum mechanics, the state of the *other, undisturbed* system will depend on what a quantity has been measured. This dependence is so strong, that one may end up with such states for the undisturbed system (in case of different measurements), that are eigenstates of *noncommuting* operators.

The violation of Bell's inequality is perhaps even more powerful as it is experimentally proven[5]. Bell's inequality refers to a situation when measurements are performed on each of the separated systems mentioned above. One assumes that any correlation between the results of the measurements performed on the different systems can come only from the previous interaction which created the entangled state. Therefore, one supposes that there are some stable properties attached to each system, so that these properties

[1] Note that this is certainly not the case in quantum cosmology. One important physical motivation for studying the fundamental questions of quantum mechanics is the hope that the correct answer will help establishing the quantum theory of gravitation.

[2] At least if one assumes that quantum mechanics is a *complete* theory (in case of the EPR paradox) and if one maintains *realism* and *inductive inference*[4] (in case of the violation of Bell's inequality).

'store' the correlation after the systems have become separated, and they determine (at least in a probabilistic sense) the outcome of the corresponding measurements. With these assumptions one finds that the correlations cannot be arbitrary but must satisfy a certain inequality. This is Bell's inequality. The correlations may be calculated quantum mechanically, and the quantum prediction *does not* always satisfy Bell's inequality. Correlations are measurable quantities, and experiments have proved the correctness of the quantum prediction and thus the violation of Bell's inequality.

What does it mean? Most people seem to believe that the above results indeed imply that separated systems can influence each other. Nevertheless, we maintain that such a conclusion is physically unacceptable. The principle of locality (or Einstein separability) has served us well in every branch of physics, even in quantum physics, including the most sophisticated quantum field theories. It is rather hard to believe that it would fail only in case of measurements. After all, a measurement is just an interaction between two physical systems, one of them being a macroscopic measuring device which consists of atoms whose structure and interactions are rather well understood in terms of quantum mechanics. There is no room for a mysterious nonlocal influence.

In case of the EPR paradox it is obvious that if one wants to maintain Einstein separability, he must change the interpretation and must replace the collapse of the wave function with something else.

The experimentally observed violation of Bell's inequality is more puzzling. The derivation of Bell's inequality is completely independent of quantum mechanics, it is based on a few very fundamental assumptions[4]: realism, inductive inference and Einstein separability. Realism and inductive inference are not less important in physics than Einstein separability, so we do not want to give up them, either. The only way out can be if there is some further, independent and hidden assumption, which seems to us obvious, but which is not valid in quantum mechanics.

In the present letter it will be shown that it is indeed the case. One may reinterpret the meaning and the interrelations of the quantum states such a manner that the EPR paradox and the violation of Bell's inequality gain a natural explanation without giving up realism, inductive inference or Einstein separability. The hidden, not allowed assumption mentioned above is connected to the fact that in the new theory the simultaneous existence of the different states is usually restricted in a particular way, namely, although each state exists separately, they cannot be compared without essentially disturbing the system. In case of the violation of Bell's inequality it turns out that the states of the measuring devices and those which 'store' the correlations are not comparable (as any attempt for a comparison changes the correlations), so the usual picture about stable properties which are comparable at any time with anything and 'store' the correlations does not apply, although the correlations may be attributed exclusively to the 'common past' (previous interaction) of the particles.

As we shall see, the theory proposed here involves a rather fundamental change of the basic concepts of quantum mechanics. Measurements will not be the primary concepts any longer, there will be no collapse of the wave function, and quantum states exhibit a new kind of dependence on *quantum reference systems*, a new concept to be explained below. On the other hand, the quantum mechanics of closed systems does not change, especially, Schrödinger's equation remains to be valid, and thus all the usual measurable consequences are unchanged. The *only* advantage of the present theory is that it respects the principle of locality and offers a consistent and physically acceptable basis for quantum mechanics.

2 The basic new concept: quantum reference systems

Let us consider a simple example, namely, an idealized measurement of an $\hat{S}_z$ spin component of a spin-$\frac{1}{2}$ particle. Be the particle P initially in the state

$$\alpha|\uparrow> +\beta|\downarrow> , \tag{1}$$

where $|\alpha|^2 + |\beta|^2 = 1$ and the states $|\uparrow>$ and $|\downarrow>$ are the eigenstates of $\hat{S}_z$ corresponding to the eigenvalues $\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$, respectively. The other quantum numbers and variables have been suppressed. The dynamics of the measurement is given by the relations $|\uparrow> |m_0> \rightarrow\ |\uparrow> |m_\uparrow>$ and $|\downarrow> |m_0> \rightarrow\ |\downarrow> |m_\downarrow>$, where $|m_0>$ stands for the state of the measuring device M (e.g. a Stern-Gerlach apparatus) before the measurement (no spot on the photographic plate), while $|m_\uparrow>$ ($|m_\downarrow>$) is the state of the measuring device after the measurement that corresponds to the measured spin value $\frac{\hbar}{2}$ ($-\frac{\hbar}{2}$). The shorthand notation $\rightarrow$ stands for the unitary time evolution during the measurement, which is assumed to fulfill the time dependent

Schrödinger equation corresponding to the total Hamiltonian of the combined $P+M$ system. As the initial state of the particle is given by Eq.(1), the linearity of the Schrödinger equation implies that the measurement process can be written as

$$(\alpha|\uparrow> +\beta|\downarrow>)|m_0> \rightarrow\ |\Psi>= \alpha|\uparrow>|m_\uparrow> +\beta|\downarrow>|m_\downarrow> \quad . \qquad (2)$$

Let us consider now the state of the measuring device M after the measurement. As the combined system $P+M$ is in an entangled state, the measuring device has no own wave function and may be described by the *reduced density matrix*[6]

$$\hat{\rho}_M = Tr_P\left(|\Psi><\Psi|\right) = |m_\uparrow>|\alpha|^2<m_\uparrow| + |m_\downarrow>|\beta|^2<m_\downarrow| \quad , \qquad (3)$$

where Tr_P stands for the trace operation in the Hilbert space of the particle P. Nevertheless, if we look at the measuring device, we certainly see that either $\frac{\hbar}{2}$ or $-\frac{\hbar}{2}$ spin component has been measured, that correspond to the states $|m_\uparrow>$ and $|m_\downarrow>$, respectively. These are obviously not the same as the state (3). Indeed, $|m_\uparrow>$ and $|m_\downarrow>$ are pure states while $\hat{\rho}_M$ is a mixture of them. Why do we get different states? According to orthodox quantum mechanics, one may argue as follows. The reduced density matrix $\hat{\rho}_M$ has been calculated from the state $|\Psi>$ (cf. Eq.(2)) of the whole system $P+M$. A state is a result of a measurement (the preparation), so we may describe M by $\hat{\rho}_M$ if we have gained our information about M from a measurement done on $P+M$. On the other hand, looking at the measuring device directly is equivalent with a measurement done directly on M. In this case M is described by either $|m_\uparrow>$ or $|m_\downarrow>$. We may conclude that performing measurements on different systems (each containing the system we want to decribe) gives rise to different descriptions (in terms of different states). Let us call the system which has been measured (it is $P+M$ in the first case and M in the second case) the *quantum reference system*. Using this terminology, we may tell that we make a measurement on the quantum reference system R, thus we prepare its state $|\psi_R>$ and using this information we calculate the state $\hat{\rho}_S(R) = Tr_{R\setminus S}|\psi_R><\psi_R|$ of a subsystem S. We shall call $\hat{\rho}_S(R)$ the state of S with respect to R. Obviously $\hat{\rho}_R(R) = |\psi_R><\psi_R|$, thus $|\psi_R>$ may be identified with the state of the system R with respect to itself.

Let us emphasize that up to now, despite of the new terminology, there is nothing new in the discussion. We have merely considered some rather elementary consequences of basic quantum mechanics.

Let us return now to the question why the state of the system S (i.e., $\hat{\rho}_S(R)$) depends on the choice of the quantum reference system R. In the spirit of the Copenhagen interpretation one would answer that in quantum mechanics measurements unavoidably disturb the systems, therefore, if we perform measurements on different surroundings R, this disturbance is different, and this is reflected in the R-dependence of $\hat{\rho}_S(R)$. Nevertheless, this argument is not compelling. We may also assume that the states of the systems have already existed before the measurements, and that there may exist measurements which do not change these states. Then the R-dependence of $\hat{\rho}_S(R)$ becomes an inherent property of quantum mechanics. Let us leave at this decisive point the traditional framework of quantum mechanics and follow the new line just sketched.

The meaning of the quantum reference systems is now analogous to the classical coordinate systems. Choosing a classical coordinate system means that we imagine what we would experience if we were there. Similarly, choosing a quantum reference system R means that we imagine what we would experience if we did a measurement on R that does not disturbe $\hat{\rho}_R(R) = |\psi_R >< \psi_R|$. In order to see that such a measurement exists, consider an operator $\hat{A}$ (which acts on the Hilbert space of R) whose eigenstates include $|\psi_R >$. The measurement of $\hat{A}$ will not disturbe $|\psi_R >$. Let us emphasize that the possibility of nondisturbing measurements is an expression of realism: the state $\hat{\rho}_R(R)$ exists independently whether we measure it or not.

As the dependence of $\hat{\rho}_S(R)$ on R is a fundamental property now, one has to specify the relation of the different states in terms of suitable postulates. Below we list these postulates.

3 Rules of the new framework

Postulate 1. *The system S to be described is a subsystem of the reference system R.*

Postulate 2. *The state $\hat{\rho}_S(R)$ is a positive definite, Hermitian operator with unit trace, acting on the Hilbert space of S.*

Definition 1. *$\hat{\rho}_S(S)$ is called the internal state of S.*

Postulate 3. *The internal states $\hat{\rho}_S(S)$ are always projectors, i.e., $\hat{\rho}_S(S) = |\psi_S >< \psi_S|$.*

In the following these projectors will be identified with the corresponding wave functions $|\psi_S>$ (as they are uniquely related, apart from a phase factor).

Postulate 4. *The state of a system S with respect to the reference system R (denoted by $\hat{\rho}_S(R)$) is the reduced density matrix of S calculated from the internal state of R, i.e. $\hat{\rho}_S(R) = Tr_{R\setminus S}\left(\hat{\rho}_R(R)\right)$, where $R \setminus S$ stands for the subsystem of R complementer to S.*

Definition 2. *An isolated system is such a system that has not been interacting with the outside world. A closed system is such a system that is not interacting with any other system at the given instant of time (but might have interacted in the past).*

Postulate 5. *If I is an isolated system then its state is independent of the reference system R: $\hat{\rho}_I(R) = \hat{\rho}_I(I)$.*

Postulate 6. *If the reference system $R = I$ is an isolated one then the state $\hat{\rho}_S(I)$ commutes with the internal state $\hat{\rho}_S(S)$.*

This means that the internal state of S coincides with one of the eigenstates of $\hat{\rho}_S(I)$.

Definition 3. *The possible internal states are the eigenstates of $\hat{\rho}_S(I)$ provided that the reference system I is an isolated one.*

Postulate 7. *If I is an isolated system, then the probability $P(S, j)$ that the eigenstate $|\phi_{S,j}>$ of $\hat{\rho}_S(I)$ coincides with $\hat{\rho}_S(S)$ is given by the corresponding eigenvalue λ_j.*

Postulate 8. *The result of a measurement is contained unambigously in the internal state of the measuring device.*

Postulate 9. *If there are n ($n = 2,\ 3,\ \ldots$) disjointed physical systems, denoted by*
$S_1, S_2, \ldots S_n$, all contained in the isolated reference system I and having the possible internal states $|\phi_{S_1,j}>, |\phi_{S_2,j}>, \ldots, |\phi_{S_n,j}>$, respectively, then the joint probability that $|\phi_{S_i,j_i}>$ coincides with the internal state of S_i ($i = 1, ..n$) is given by

$$\begin{aligned} &P(S_1, j_1, S_2, j_2, \ldots, S_n, j_n) \\ &= Tr_{S_1+S_2+\ldots+S_n}[\hat{\pi}_{S_1,j_1}\hat{\pi}_{S_2,j_2}\ldots\hat{\pi}_{S_n,j_n}\hat{\rho}_{S_1+S_2+\ldots+S_n}(I)], \end{aligned} \tag{4}$$

where $\hat{\pi}_{S_i,j_i} = |\phi_{S_i,j_i}><\phi_{S_i,j_i}|$.

Postulate 10. *The internal state $|\psi_C>$ of a closed system C satisfies the time dependent Schrödinger equation $i\hbar\partial_t|\psi_C> = \hat{H}|\psi_C>$.*

Here $\hat{H}$ stands for the Hamiltonian.

It is an important feature of the theory that the states defined with respect to different quantum reference systems are not necessarily comparable. What does it mean? As we have seen one may determine the state $\hat{\rho}_R(R)$ of any system R without disturbing it if one performs a suitable measurement on R. We may do it with *one* arbitrarily chosen system R. But can we do this with *two* (or more) systems R_1, R_2 (R_3, ...) at the same time (or in succession)? In other terms: may we always attribute a physical meaning to the *simultaneous* existence of $\hat{\rho}_{R_1}(R_1)$ and $\hat{\rho}_{R_2}(R_2)$? The answer is no. If we perform a nondisturbing measurement on R_1, this will disturbe $\hat{\rho}_{R_2}(R_2)$ except when $R_2 \subseteq R_1$ or $R_1 \cap R_2 = \emptyset$. Let us demonstrate this phenomenon on a simple example. Consider two different spin-$\frac{1}{2}$ particles P_1 and P_2 and be $R_1 = P_1$, $R_2 = P_1 + P_2$. Suppose that $P_1 + P_2$ is initially an isolated system and

$$|\psi_{P_1+P_2}>= a|1,\uparrow>|2,\downarrow> -b|1,\downarrow>|2,\uparrow> \quad , \tag{5}$$

where $|a|^2 + |b|^2 = 1$. The notation $|1,\uparrow>$ stands for such a state of the first particle, where the z component of the spin (denoted by $\hat{S}_{1z}$) has the definite value $+\frac{\hbar}{2}$. The other notations have an analogous meaning. Further, suppose that $|\psi_{P_1}>=|1,\uparrow>$. Measuring $\hat{S}_z$ on P_1 we may write

$$|\psi_{P_1+P_2}>|m_0> \rightarrow \; a|1,\uparrow>|2,\downarrow>|m_\uparrow> -b|1,\downarrow>|2,\uparrow>|m_\downarrow> \quad , \tag{6}$$

where $|m_0>$, $|m_\uparrow>$ and $|m_\downarrow>$ are the states of the measuring device M as defined earlier. Eq.(6) implies

$$\begin{aligned}\hat{\rho}_{P_1+P_2}(P_1+P_2+M) = &|1,\uparrow>|2,\downarrow>|a|^2<2,\downarrow|<1,\uparrow| \\ &+|1,\downarrow>|2,\uparrow>|b|^2<2,\uparrow|<1,\downarrow|\end{aligned} \tag{7}$$

thus $|\psi_{P_1+P_2}>$ has changed and has become due to the measurement either $|1,\uparrow>|2,\downarrow>$ or $|1,\downarrow>|2,\uparrow>$.

We may summarize this situation as follows: all the states $\hat{\rho}_S(R)$ exist *individually* (where R and $S \subset R$ may be any existing system), as we may choose a system R at will and may perform a suitable measurement on it in order to learn the states $\hat{\rho}_S(R)$ (now R is fixed) without changing them. Nevertheless, once we perform this measurement we unavoidably disturb the states $\hat{\rho}_{\tilde{S}}(\tilde{R})$ when $\tilde{R} \not\subseteq R$ and $\tilde{R} \cap R \neq \emptyset$, thus preventing us from learning these latter states. Therefore, no physical meaning may be attributed to the *simultaneous* existence of the states $\hat{\rho}_S(R)$ and $\hat{\rho}_{\tilde{S}}(\tilde{R})$ although they do exist *separately*.

One may perhaps think that the above property questions the reality of the states or the realism of the theory. Let us consider a simple classical analogue demonstrating this is not the case. It is known that general relativity allows coordinate systems even inside of a black hole. Consider now two different black holes and introduce a coordinate system inside each of them. Probably no one doubts the reality of the descriptions with respect to these coordinate systems. We may indeed check what we may experience with respect to *one* of these systems. We may freely choose one of the black holes and may fall into it. Then we see what is inside. However, if we do so we cannot come back and thus automatically prevent ourselves from learning the other blackhole interieur. Therefore, similarly to the quantum case, no physical meaning can be attributed to the *simultaneous* existence of descriptions with respect to the above two coordinate systems. Of course, we do not want to say that there is any deeper connection between the underlying physics of the quantum case and that of the above classical example.

Let us mention one more unusual feature of the quantum reference systems, namely, that the states of the same system with respect to different quantum reference systems are not uniquely related. Indeed, in our first example (cf. Eq.(3)) $\hat{\rho}_M(P+M) = |m_\uparrow > |\alpha|^2 < m_\uparrow| + |m_\downarrow > |\beta|^2 < m_\downarrow|$, while $\hat{\rho}_M(M)$ can be either $|m_\uparrow >$ (with probability $|\alpha|^2$) or $|m_\downarrow >$ (with probability $|\beta|^2$). Certainly this feature is an expression of the indeterministic nature of quantum mechanics.

Let us mention finally that, strictly speaking, the whole formalism of the present theory is connected to the experience only via Postulate 8. One can indeed see that measurement is not the primary concept any longer. The theory works such a way that - on the basis of the results of previous measurements - one assumes an initial state of an isolated system which includes the present measuring device as well, calculates the final state from the Schrödinger equation and finally, using the Postulates, deduces the state $\hat{\rho}_M(M)$ of the measuring device M to get a prediction concerning the outcome of the measurement. Certainly, this prediction will be usually probabilistic.

4 Solution of the EPR paradox

Consider again the two-particle system $P_1 + P_2$ (cf. Eq.(5)). Suppose one performs a measurement on the first particle. Let us consider the situation when one measures $\hat{S}_{1z'}$, where the z' axis is obtained from the z axis by a rotation at an angle δ around the x axis. The initial state of the whole system (including the measuring device) is given by $|m_0>(a|1,\uparrow>|2,\downarrow> -b|1,\downarrow>|2,\uparrow>)$, where $|m_0>$ stands for the initial state of the measuring device. The time evolution during the measurement can be established by using the relations $|m_0>|1,\delta,\uparrow> \to |m_+>|1,\delta,\uparrow>$ and $|m_0>|1,\delta,\downarrow> \to |m_->|1,\delta,\downarrow>$, where $|1,\delta,\uparrow>=\cos(\frac{\delta}{2})|1,\uparrow> -\sin(\frac{\delta}{2})|1,\downarrow>$, and $|1,\delta,\downarrow>=\sin(\frac{\delta}{2})|1,\uparrow> +\cos(\frac{\delta}{2})|1,\downarrow>$. Thus the final state of the whole system is

$$|m_+>|1,\delta,\uparrow>\left(a\,\cos(\frac{\delta}{2})|2,\downarrow> +b\,\sin(\frac{\delta}{2})|2,\uparrow>\right)$$
$$+|m_->|1,\delta,\downarrow>\left(a\,\sin(\frac{\delta}{2})|2,\downarrow> -b\,\cos(\frac{\delta}{2})|2,\uparrow>\right) \quad . \tag{8}$$

According to the Copenhagen interpretation one ought to apply the concept of the reduction of the wave function, which yields that the state of the second, *undisturbed* particle has the state (after the measurement)

$$|\varphi_+^{(2)}>=$$
$$\left(|a|^2\cos^2(\frac{\delta}{2})+|b|^2\sin^2(\frac{\delta}{2})\right)^{-\frac{1}{2}}\left(a\,\cos(\frac{\delta}{2})|2,\downarrow> +b\,\sin(\frac{\delta}{2})|2,\uparrow>\right) \quad , \tag{9}$$

if we have measured $\frac{\hbar}{2}$ and

$$|\varphi_-^{(2)}>=$$
$$\left(|a|^2\sin^2(\frac{\delta}{2})+|b|^2\cos^2(\frac{\delta}{2})\right)^{-\frac{1}{2}}\left(a\,\sin(\frac{\delta}{2})|2,\downarrow> -b\,\cos(\frac{\delta}{2})|2,\uparrow>\right) \quad , \tag{10}$$

if we have measured $-\frac{\hbar}{2}$. These states depend on δ, i.e., on the quantity $\hat{S}_{1z'}$ which has been measured on the first particle.

According to the present theory there is no collapse of the wave function, however, the states (9) and (10) may be easily identified by $\hat{\rho}_{P_2}(P_1 + P_2)$.

Indeed,

$$
\begin{aligned}
&\hat{\rho}_{P_1+P_2}(P_1+P_2+M) \\
&= |1,\delta,\uparrow>|\varphi_+^{(2)}> \left(|a|^2\cos^2(\frac{\delta}{2})+|b|^2\sin^2(\frac{\delta}{2})\right) <\varphi_+^{(2)}|<1,\delta,\uparrow| \\
&\quad +|1,\delta,\downarrow>|\varphi_-^{(2)}> \left(|a|^2\sin^2(\frac{\delta}{2})+|b|^2\cos^2(\frac{\delta}{2})\right) <\varphi_-^{(2)}|<1,\delta,\downarrow|\ .
\end{aligned}
$$

Its eigenstates are

$$|1,\delta,\uparrow>|\varphi_+^{(2)}> \tag{11}$$

and

$$|1,\delta,\downarrow>|\varphi_-^{(2)}>\ \ . \tag{12}$$

Thus $\hat{\rho}_{P_1+P_2}(P_1+P_2) = |\psi_{P_1+P_2}><\psi_{P_1+P_2}|$ where $|\psi_{P_1+P_2}>$ coincides with either (11) or (12). Calculating $\hat{\rho}_{P_2}(P_1+P_2) = Tr_{P_1}(\hat{\rho}_{P_1+P_2}(P_1+P_2))$ we arrive at the expressions $|\varphi_+^{(2)}><\varphi_+^{(2)}$ or $|\varphi_-^{(2)}><\varphi_-^{(2)}|$ (cf. Eqs.(9),(10), respectively). The dependence on δ is now easily understood: the measurement done on P_1 influences the quantum reference system P_1+P_2, hence $\hat{\rho}_{P_2}(P_1+P_2)$ depends on the measurement, although P_2 has not been influenced. Certainly, Einstein separability is not violated.

Einstein separability requires now that $\hat{\rho}_{P_2}(P_2)$ must be independent of the measurement done on P_1. Direct calculation shows that $\hat{\rho}_{P_2}(P_1+P_2+M)$ is the same both before and after the measurement, thus its eigenstates are unchanged, too. Therefore, $\hat{\rho}_{P_2}(P_2)$ is not influenced by the measurement. One may also prove[7] that quite generally, if the system A is separated from the systems B and C, $\hat{\rho}_A(A)$ is independent of the interaction between B and C.

5 Explanation of the violation of Bell's inequality

Let us consider again the previous two-particle system. In order to exhibit the mathematical structure we write the state (5) as

$$\sum_j c_j|\phi_{P_1,j}>|\phi_{P_2,j}> \tag{13}$$

where $c_1 = a$, $c_2 = -b$, $|\phi_{P_1,1}>=|1,\uparrow>$, $|\phi_{P_1,2}>=|1,\downarrow>$, $|\phi_{P_2,1}>=|2,\downarrow>$, $|\phi_{P_2,2}>=|2,\uparrow>$. When the two particle system is in the state (13), there are strong correlations between the states $\hat{\rho}_{P_1}(P_1) = |\psi_{P_1}><\psi_{P_1}|$ and $\hat{\rho}_{P_2}(P_2) = |\psi_{P_2}><\psi_{P_2}|$. Provided that the system P_1+P_2 is initially isolated, applying **Postulate 9** we obtain that the probability that $|\psi_{P_1}>=|\phi_{P_1,j}>$ and $|\psi_{P_2}>=|\phi_{P_2,k}>$ is $P(P_1,j,P_2,k) = |c_j|^2\delta_{j,k}$.

Let us consider now a typical experimental situation, when measurements on both particles are performed. We shall show that according to the present theory the observed correlations are exclusively due to the previous interaction between the particles. Before the measurements the internal state of the isolated system $P_1+M_1+P_2+M_2$ (P_1, P_2 stands for the particles and M_1, M_2 for the measuring devices, respectively) is given by $\left(\sum_j c_j|\phi_{P_1,j}>|\phi_{P_2,j}>\right)|m_0^{(1)}>|m_0^{(2)}>$, while it is

$$\sum_j c_j\hat{U}_t(P_1+M_1)\left(|\phi_{P_1,j}>|m_0^{(1)}>\right)\hat{U}_t(P_2+M_2)\left(|\phi_{P_2,j}>|m_0^{(2)}>\right) \quad , (14)$$

a time t later, i.e. during and after the measurements. Here $\hat{U}_t(P_i+M_i)$ ($i=1,2$) stands for the unitary time evolution operator of the closed system P_i+M_i.

Eq.(14) implies that the internal states of the closed systems P_1+M_1 and P_2+M_2 evolve unitarily and do not influence each other. This time evolution can be given explicitly through the relations

$$|\xi(P_i,j)>|m_0^{(i)}> \rightarrow\ |\xi(P_i,j)>|m_j^{(i)}> \quad , \tag{15}$$

where $i,j=1,2$ and $|\xi(P_i,j)>$ is the j-th eigenstate of the spin measured on the i-th particle along an axis $z^{(i)}$ which closes an angle ϑ_i with the original z direction. The time evolution of the internal state of the closed systems P_i+M_i is given explicitly by $|\psi_{P_i}>|m_0^{(i)}>\rightarrow\ \sum_j<\phi_{P_i,j}|\psi_{P_i}>|\phi_{P_i,j}>|m_j^{(i)}>$. As we see, the i-th measurement process is completely determined by the initial internal states of the particle P_i. Therefore, any correlation between the measurements may only stem from the initial correlation of the internal states of the particles.

For the calculation of the state $\hat{\rho}_{M_1}(M_1)$ (which corresponds to the measured value) one needs to know the state of the whole isolated system P_1+

$P_2 + M_1 + M_2$. Using Eq.(15) the final state (14) may be written as

$$\sum_{j,k} \left(\sum_l c_l < \xi(P_1, j)|\phi_{P_1,l} >< \xi(P_2, k)|\phi_{P_2,l} > \right)$$
$$\times |m_j^{(1)} > |m_k^{(2)} > |\xi(P_1, j) > |\xi(P_2, k) > .$$

Direct calculation shows that

$$\hat{\rho}_{M_1}(P_1 + P_2 + M_1 + M_2)$$
$$= \sum_j \left(\sum_l |c_l|^2 | < \xi(P_1, j)|\phi_{P_1,l} > |^2 \right) |m_j^{(1)} >< m_j^{(1)}|.$$

Note that it is independent of the second measurement.

According to **Postulate 6** $|\psi_{M_1} >$ is one of the $|m_j^{(1)} >$-s. (Similarly one may derive that $|\psi_{M_2} >$ is one of the $|m_k^{(2)} >$-s.) The probability of the observation of the j-th result (up or down spin in a chosen direction) is

$$P(M_1, j) = \sum_l |c_l|^2 | < \xi(P_1, j)|\phi_{P_1,l} > |^2 \; . \tag{16}$$

This may be interpreted in conventional terms: $|c_l|^2$ is the probability that $|\psi_{P_1} >= |\phi_{P_1,l} >$, and $| < \xi(P_1, j)|\phi_{P_1,l} > |^2$ is the conditional probability that one gets the j-th result if $|\psi_{P_1} >= |\phi_{P_1,l} >$. Thus we see that the initial internal state of P_1 determines the outcome of the first measurement in the usual probabilistic sense. One may show quite similarly that the initial internal state of P_2 determines the outcome of the second measurement in the same way.

But doesn't it mean that the internal states of P_1 and P_2 play the role of local hidden variables? Not at all, because hidden variables are thought to be comparable with the results of the measurements so that their joint probability may be defined, while in our theory there is no way to define the joint probability $P(P_1, l_1, P_2, l_2, (0); M_1, j, M_2, k, (t))$, i.e., the probability that initially $|\psi_{P_1} >= |\phi_{P_1,l_1} >$ and $|\psi_{P_2} >= |\phi_{P_2,l_2} >$ *and* finally $|\psi_{M_1} >= |m_j^{(1)} >$ and $|\psi_{M_2} >= |m_k^{(2)} >$. Intuitively we would write

$$P(P_1, l_1, P_2, l_2, (0); M_1, j, M_2, k, (t))$$
$$= |c_{l_1}|^2 \delta_{l_1,l_2} | < \xi(P_1, j)|\phi_{P_1,l_1} > |^2 | < \xi(P_2, k)|\phi_{P_2,l_2} > |^2 \; , \tag{17}$$

as $|c_{l_1}|^2\delta_{l_1,l_2}$ is the joint probability that $|\psi_{P_1} >= |\phi_{P_1,l} >$ and $|\psi_{P_2} >= |\phi_{P_2,l} >$, and $| < \xi(P_i,j)|\phi_{P_i,l_i} > |^2$ is the conditional probability that one gets the j-th result in the $i-th$ measurement if initially $|\psi_{P_i} >= |\phi_{P_i,l_i} >$ $(i = 1,2)$. Certainly the existence of such a joint probability would immediately imply the validity of Bell's inequality, thus it is absolutely important to understand why this probability does not exist.

Let us mention, first of all, that using **Postulate 9** for $n = 2$, we may calculate the correlation between the measurements, i.e., the joint probability that $|\psi_{M_1} >= |m_j^{(1)} >$ *and* $|\psi_{M_2} >= |m_k^{(2)} >$. We obtain

$$P(M_1,j,M_2,k) = \left|\sum_l c_l < \xi(P_1,j)|\phi_{P_1,l} >< \xi(P_2,k)|\phi_{P_2,l} >\right|^2 \quad . \tag{18}$$

This is the usual quantum mechanical expression which violates Bell's inequality and whose correctness is experimentally proven. Thus our theory gives the correct expression for the correlation. Nevertheless, if the joint probability (17) exists, it leads to

$$P(M_1,j,M_2,k) = \sum_l |c_l|^2| < \xi(P_1,j)|\phi_{P_1,l} > |^2| < \xi(P_2,k)|\phi_{P_2,l} > |^2 \tag{19}$$

which satisfies Bell's inequality and contradicts Eq.(18). Let us demonstrate that no such contradiction appears.

Evidently, the joint probability $P(P_1,l_1,P_2,l_2,(0);M_1,j,M_2,k,(t))$ can be physically meaningful only if one can compare the initial internal states of P_1 and P_2 with the final internal states of M_1 and M_2 by suitable nondisturbing measurements. It turns out, however, that any attempt for such a comparison influences the system so strongly that the correlations $P(M_1,j,M_2,k)$ change.

If we try to compare the initial internal states of P_1 and of P_2 with the final internal states of M_1 and M_2, the first difficulty appears because we want to compare states given at different times. Nevertheless, as the initial internal state of P_i is uniquely related to the final internal state of the system P_i+M_i, the joint probability $P(P_1,l_1,P_2,l_2,(0);M_1,j,M_2,k,(t))$ (if exist) coincides with $P(P_1+M_1,l_1,P_2+M_2,l_2,M_1,j,M_2,k)$, where all the occuring states are given after the measurements. As the systems $P_1+M_1,\ P_2+M_2,\ M_1,\ M_2$ are not disjointed, our **Postulates** do not provide us with an expression for the joint probability we are seeking for. If we check $|\psi_{M_1} >$ and $|\psi_{M_2} >$ by suitable nondisturbing measurements, we destroy $|\psi_{P_1+M_1} >$ and $|\psi_{P_2+M_2} >$ (cf.

the discussion at the end of Section 2.), inhibiting any comparison. On the other hand, if we check first $|\psi_{P_1+M_1}>$ and $|\psi_{P_2+M_2}>$, then $P(M_1, j, M_2, k)$ changes.[3] Indeed, after suitable measurements performed on $P_i + M_i$ (which do not change the internal states of $P_i + M_i$)[4] by further measuring devices $\tilde{M}_i$ we get for the internal state of the whole system

$$\sum_l c_l \left(\sum_j < \xi(P_1, j) | \phi_{P_1,l} > | \xi(P_1, j) > | m_j^{(1)} > \right)$$
$$\times \left(\sum_k < \xi(P_2, k) | \phi_{P_2,l} > | \xi(P_2, k) > | m_k^{(2)} > \right) | \tilde{m}_l^{(1)} > | \tilde{m}_l^{(2)} > \ . \tag{20}$$

As the systems $M_1,\ M_2,\ \tilde{M}_1,\ \tilde{M}_2$ are disjointed, we may apply **Postulate 9** for $n = 4$ and we indeed get for $P(\tilde{M}_1, l_1, \tilde{M}_2, l_2, M_1, j, M_2, k)$ the expression (17). Do we get then a contradiction with Eq.(18)? No, because applying **Postulate 9** for $n = 2$ directly, we get in this case Eq.(19) instead of Eq.(18). Thus we see that the extra measurements have changed the correlations and our theory gives account of this effect consistently.

Summarizing, we have seen that the initial internal state of P_1 (P_2) determines the first (second) measurement process, therefore, these states 'carry' the initial correlations and 'transfer' them to the measuring devices. As the measurement processes do not influence each other, the observed correlations may stem only from the 'common past' of the particles. On the other hand, any attempt to compare the initial internal states of P_1 and P_2 with the results of both measurements changes the correlations, thus a joint probability for the simultaneous existence of these states cannot be defined. This means that the reason for the violation of Bell's inequality is that the usual derivations always assume that the states (or 'stable properties') which carry the initial correlations can be freely compared with the results of the measurements. This comparability is usually thought to be a consequence of realism. According to the present theory, the above assumption goes beyond the requirements of realism and proves to be wrong, because each of

[3] Therefore, in the original situation the joint probability $P(P_1 + M_1, l_1, P_2 + M_2, l_2, M_1, j, M_2, k)$ cannot be defined at all. Here we are faced with the feature of our theory discussed at the end of Section 2: although states with respect to different quantum reference systems do exist individually, their simultaneous occurence or comparatibility may not be defined.

[4] This is equivalent by recording the initial internal state of P_i.

the states $|\psi_{P_1+M_1}>$, $|\psi_{P_2+M_2}>$, $|\psi_{M_1}>$ and $|\psi_{M_2}>$ exists individually, but they cannot be compared without changing the correlations.

In conclusion, it has been shown that by suitable redefinition of the physical meaning of the quantum states and their interrelations one can solve the EPR paradox and can explain the violation of Bell's inequality without giving up realism, inductive inference or Einstein separability.

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