

# Olbers' Paradox

## A Review of Resolutions to this Paradox

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### **Abstract**

In a homogeneous Universe, infinite in space and time, every line of sight will end on the surface of a star. So why is the sky dark at night?

This is the question posed by Heinrich Olbers in 1826, although the problem had been around since 1577. This essay examines the various solutions proposed over the last five hundred years and reveals the cosmological significance of a dark night sky. The story of Olbers Paradox is the story of our evolving view of the Universe.

Before the sixteenth century the western world's model of the universe hadn't changed since Aristotle first proposed it nearly two thousand years earlier. This model consisted of the Earth at the centre of the universe, with the sun and planets revolving round it. Enclosing the entire solar system was a spherical shell containing the stars and beyond this shell was the 'void'. As a scientific theory this model agreed with all the evidence available at the time, both observational and theological, and the idea of a finite, bounded Universe centred on the Earth was accepted as self evident. The only part of this geocentric model that had been questioned was the exact path that the planets followed as they circled the Earth - originally thought to be perfect circles the paths had evolved into highly convoluted loops, sometimes doubling back on themselves, which still failed to predict the future motion of the planets. In an attempt to return to the perfectly circular orbits envisioned by the ancient Greeks, the Polish astronomer Nicolaus Copernicus introduced a new heliocentric model of the Universe. This model placed the sun at the centre of the (finite, bounded) Universe with the planets circling it, and unfortunately proved no better than the geocentric model in predicting the motion of the planets - but by removing the Earth from the centre of the Universe Copernicus had laid the foundations of a controversial scientific principle that would be fiercely debated for the next five hundred years and now forms the backbone of all modern cosmology.

The implicit argument which Copernicus (probably unknowingly) introduced in his model is this: If the Earth is not at the centre of the Universe, why should any place be at the centre of the Universe or why should the Universe even have a centre? This idea was not actually a new one. The epic poem 'The Nature of the Universe' written by Lucretius in 55 b.c. had recently been discovered and the idea of an infinite Universe was known, if not seriously considered, by many writers of the period.

One of the first people to seriously suggest an infinite Universe was the mathematician and astronomer Thomas Digges who published his 'Perfect Description of the Celestial Spheres' in London in 1576. Digges boldly dismantled the Aristotelian sphere of fixed stars and randomly scattered the stars throughout infinite space. In doing this he noticed a problem with his model which forms the earliest description of what is now known as Olbers' Paradox: Why did the now infinite number of stars not make the night sky bright? The answer, said Digges, is that most of them are too far away to be seen.

Although Digges included this solution only as a 'throw away' line in his book, the paradox has captured the imagination of astronomers ever since. Other 'infinite Universe' proposers of the sixteenth and seventeenth centuries came to the same conclusion, even though Digges solution doesn't stand up to an even rudimentary investigation: even if the

stars were too far away to be seen individually the collective flux of light from all the stars would cause the night sky to glow.

The first person to realise this was Johannes Kepler, who with brilliant intuition noted that the relative darkness of the night sky is of huge cosmological significance. In his 1610 pamphlet 'Conversation with the Starry Messenger' Kepler argued for a finite, bounded Universe by showing that in an infinite Universe, with stars scattered through space the 'whole celestial vault would be as luminous as the sun.'

Kepler's devastating argument would seem to have disproved the Infinite Universe theory, but following the work of Galileo and, ironically, Kepler himself, which confirmed the Copernican Heliocentric model the idea of a bounded Universe seemed less and less attractive to many astronomers. Isaac Newton's recently published Laws of Gravitation which were unbelievably successful in explaining and predicting the planetary motions also required an infinite homogeneous Universe (to prevent the Universe collapsing in on itself), and so the solving of the paradox became a priority with nearly every astronomer of the next 300 years, turning his attention to it.

The first serious mathematical analysis of the solution was carried out by the Swiss mathematician Jean Phillipe Loys de Chesaux in 1744. Drawing on the work of Edmund Halley (who performed a similar analysis to de Chesaux but unaccountably reached the same conclusion as Digges) de Chesaux constructed a series of large imaginary concentric shells of uniform thickness with the observer located at the centre. If the thickness of each shell is much smaller than the radius (the distance to the observer) then the number of stars in any shell is proportional to its volume which is proportional to the square of its radius, but the light received at the centre from any star is inversely proportional to the square of the radius. In this way he showed that the proportion of sky covered by stars is the same for every shell. De Chesaux then added shells out to a distance of 3 thousand trillion (!) light years and showed at this 'background' distance the sky became fully covered by stars (approximately  $1 \times 10^{46}$  of them). As the whole sky is 180,000 times larger than the sun's disk, the total starlight falling on the Earth should be 180,000 times more intense than sunlight. Perhaps overcome by the enormity of the problem de Chesaux then feebly suggested that there is some interstellar absorbing medium which attenuates the starlight - before, no doubt, going to lie down in a darkened room.

Despite the unlikely solution offered by de Chesaux no-one would come up with anything better for over 150 years. In 1826 Heinrich Olbers performed a similar (although less mathematically rigorous) treatment to de Chesaux and came to a similar solution. Despite merely re-hashing Halley's and de Chesaux's work the modern form of the paradox is named after him, perhaps because he offered the most succinct version of it by introducing

the line of sight argument: In an infinite, homogeneous Universe every line of sight will end upon the surface of a Star. So why is the sky dark at night? The advantage offered by this argument is that it doesn't require the stars to be randomly scattered in space, but also works if the stars are grouped in clumps (i.e. galaxies).

It was only five years later in 1831 that John Herschel laid to rest the attenuating medium theory by showing that in a Universe filled with radiation 180,000 times more intense than sunlight the Earth itself would vaporise within a few hours - clearly any absorbing medium in interstellar space would soon heat up and cease to absorb, it would then merely diffuse the radiation.

Following Herschel's work no-one else suggested absorption as a solution to the paradox - with the notable exception of Edward Fournier d'Albe who suggested that although the sky is covered by stars, most of them are non-luminous (making his suggestion the earliest known theory of the existence of Black Holes, perhaps?). Another tongue-in-cheek solution of d'Albe was that the stars are not randomly scattered in space but are arranged in straight lines with more distant stars 'hiding' behind the nearby stars. Although this solution was obviously proposed in jest it illustrates a subtle change in the way the paradox was being interpreted. As Edward Harrison points out in his book 'Darkness at Night', before Herschel's work the paradox had been seen as a question of missing starlight. After Herschel, due to the lack of any realistic answers as to where the starlight had gone, the question changed to where have all the stars gone?

Despite assuming an infinite, homogeneous Universe, a popular model of the Universe in the late nineteenth century was of an immense but finite collection of stars, the Milky Way, beyond which stretched an endless void. The Victorian Universe was beginning to regress back to the Aristotelian model.

One peculiar feature of Olbers' Paradox is that despite it receiving the attention of so many famous scientists since its conception in the sixteenth century, very few treatments contained any rigorous mathematical models (de Chesaux being a notable exception). The definitive treatment of Olbers Paradox came in 1901 when Lord Kelvin published a paper 'On Ether and Gravitational Matter through Infinite Space' he showed that according to the standard (Victorian) model of his time the galaxy contained insufficient stars to cover the night sky. He then went further and showed that even if the stars stretched away through space, filling an infinite Universe, the visible stars would still fail to cover the sky. He did this by calling on his previous work showing that stars cannot shine indefinitely - the stars lifetime is limited by its available energy resources - and making the crucial step of thinking of distances to stars in terms of light travel times. Ole Roemer had shown the speed of light was finite in 1676, so it seems astounding now that no-one had made this connection

before. (Actually both Mark Twain and Edgar Allan Poe had previously written that one resolution to Olbers' Paradox was that the Universe was finite - but, being 'mere' writers, they were both ignored by scientists of the time.)

As we look out into space we also look back in time so the darkness we see between distant stars is the darkness that existed before the birth of luminous stars. Modern estimates of the background distance give a value of  $10^{23}$  light years, meaning that to see a star on every line of sight the stars must have been shining for at least  $10^{23}$  years, but the lifetime of a sun-like star is only  $10^{10}$  years. So in answer to the question where has all the starlight gone, Kelvin replies it hasn't reached us yet

Unfortunately Kelvin's paper received little attention at the time and was roundly ignored until it's 'rediscovery' by Edward Harrison in 1985.

With the start of the twentieth century came huge advances in telescope manufacture which led to a correspondingly large increase in the information about the Universe. The Victorian single galaxy model of the Universe was discarded in favour of the multi-galaxy model, and in 1916 Einstein's theory of general relativity was completed. In 1922 the Russian physicist Alexander Friedmann found a set of solutions to Einstein's equations that allowed for an expanding Universe and in 1929 Edward Hubble showed that the Universe was indeed expanding. The idea of an infinite static Universe had to be abandoned, and surprisingly Olbers Paradox played a part in determining a new model of the Universe.

By 1950 two conflicting models of the Universe had been proposed. The 'Big Bang' model proposed by Friedmann and Georges Lemaitre of Belgium suggested an expanding Universe of finite age while the 'Steady State' model of Hermann Bondi, Thomas Gold, and Fred Hoyle suggested an expanding Universe of infinite age. Both of these models relied on the Cosmological principle which was formally stated by Edward Milne in 1933. In its simplest form this states that there is no preferred place in the Universe, an idea which, as has been mentioned, had become increasingly accepted ever since Copernicus proposed the heliocentric system. Two implicit features of this principle are that the Universe is both isotropic and homogeneous i.e. on a suitably large scale the Universe is not only the same at every point in space but it also looks the same in every direction. Both of these features result in an infinite Universe.

In his 1957 essay 'Theories of Cosmology' Bondi described both theories and re-evaluated Olbers treatment of his 'dark night sky' paradox. His treatment can be summarised as follows:

Olbers four assumptions were:

- 1) The Universe is homogeneous - so the average distance between stars and the average luminosity of a star is constant throughout space.
- 2) The Universe is unchanging in time when viewed on a large enough scale.
- 3) There are no major systematic motions in the Universe
- 4) All known laws of physics apply

This set of assumptions leads to Olbers Paradox - a night sky shining as brightly as the surface of the sun - a result which is obviously untrue, so which of these assumptions can most easily be dropped?

Assumption 1) (homogeneity) has a considerable amount of observational evidence to back it up, and it would be pointless to regard assumption 4) as false, so these two assumptions should be retained.

Assumption 2) could be dropped, as if the Universe is not unchanging, one could postulate the stars only began to shine a finite time ago.

Assumption 3) could also be dropped. If the distant stars are moving rapidly away from us any light they emit could be shifted by the Doppler effect to wavelengths outside the visual. Therefore the most distant stars would be invisible to us.

As the expansion of the Universe had already been discovered and therefore assumption 3) is now known to be false, Bondi argued that this expansion alone is enough to resolve Olbers Paradox, while the Big Bang theory requires assumptions 2) and 3) be false. He also argued that in a Universe which changed in time it was likely that some features of our physical laws (especially the natural constants - the gravitational constant, the speed of light etc.) would change, requiring any cosmology to allow for all possible 'laws of nature'. In order to avoid these complications Bondi suggested that assumption 2) (time uniformity) could be kept whilst dropping assumption 3) (no expansion) which he explained by using the analogy of a swiftly flowing river. All the individual water molecules are moving but on a large scale the river looks the same at all times.

Unfortunately this theory meant abandoning the law of conservation of matter. If the Universe is unchanging in time, the average density of the Universe must also be constant, and as the Universe is known to be expanding, matter must be continuously created to 'top up' the average density. Bondi explains this by saying that the mean rate of creation would be equivalent to 'one atom of hydrogen per quart volume' (this was in 1957) every few thousand million years, which is such a low rate that it wouldn't 'conflict with the experience on which the law of conservation of matter is based.' Subsequent calculations by Edward Harrison showed that Bondi's redshift solution would only be true in a 'steady state' Universe, not in a 'big bang' finite-age Universe.

Bondi's dramatic solution to Olbers' Paradox - that the darkness of the sky is due to the expansion of the Universe had widespread appeal but in 1965 the discovery by Arno Penzias and Robert Wilson of the cosmic microwave background radiation, made clear that we live in a big bang Universe that originated in a hot, dense state. Our Universe is not bathed in solar-intensity radiation not because of the red shift, but because the Universe is young. Stars have been luminous for only 10 billion years and not enough starlight has been emitted to make the night sky bright. Lord Kelvin's solution is the correct one, for if the night sky is dark in a static Universe it is even darker in an expanding Universe due to the (small) red shift effect.

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