SAMUEL BONAYA BUYA (Login sbonayab)
Owner
Posted Apr 12, 2014 11: 52 AM

SOLUTION OF QUINTIC EQUATIONS AND HIGHER DEGREE POLYNOMIALS

AUTHOR: SAMUEL BONAYA BUYA

ABSTRACT
In this paper I explore methods of solving higher degree polynomial

KEY WORD
Methods of solving higher degree polynomials; Radical solution of the Bring
− Jerrard quintic sextic and septic equations; converting higher degree polynomials to reducible and solvable forms; three dimensional space of Bring
− Jerrard quintic equation

INTRODUCTION

Previous efforts by Leonard Euler and others to extend methods used in solving lower degree polynomials to the quintic to obtain its general solution have prove
In 1683 Ehreinfried Walther Count of Tchirnhaus, attempted at to solve the general quintic by suggesting a transformation that would eliminate some intermed
In 1786 the Swedish mathematician, Erland Samuel Bring [3 &4]discovered a transformation of a higher degree that would reduce the general quintic to what is
− Jerrard form.
Glashan [5], Young, Runge and others established that some forms of the Bring Jerrard Quintic can be solved in radicals.
In 2011 Edward Thabo Motlotle [6], in his Masters Degree desertion, established a formula for solving a Bring Jerrard quintic by assuming a solution of the form
$= K^{\frac{1}{5}} + L^{\frac{1}{5}}$
$+ M^{\frac{1}{5}}$and then converting this algebraic form into a differential form as a more secure way of obtaining fault free solution to the polynomial. The contribution
In my published articles [1], [2]I established a form in which quintics can be written for easy solvability. In subsequent papers I tried to write establish better app
In this paper I will highlight on other methods of establishing various general forms in which quintic equations and higher degree polynomial equations can be w

METHOD OF SOLVING QUINTIC EQUATIONS

In the paper titled a calculator for polynomial equation calculator for degree five and above [1]a method was highlighted in which a polynomial of degree n was
-1 factors over all rational. In this paper I will highlight on other possible methods of solving polynomials.
Consider an infinite set, $\varphi_i(x)$, of fifth degree polynomials. In the set consider a quintic f
$\in \varphi_i(x)$such that f is divisible by lower degree polynomial g; that is: $f(x) = g(x)r(x)$where $r(x)$is a polynomial, then the quintic equation $f(x)$
$= 0$ is solvable since it is reducible or relatively composite. Consider another form of an infinite set, $\psi_j(x)$of fifth degree polynomial with a set of surjective fun
For the purpose of this research I will coin a definition
Definition 1: a polynomial, f, is relatively composite if it can decomposed to the form $f(x) = g(x)r(x)$where $g(x)$and $r(x)$are lower degree polynomials.
Definition 2: two polynomials are said to be equal if their corresponding coefficients are equal.

Proposition 1: *a general quintic can have more than one general form all of which are equal as long as their corresponding coefficients are equal.*
Proposition 2: *If a general monic quintic is in general an irreducible monic polynomial in its various general forms then it has no general solution. If it is reduci*
− existent.
Proposition 3: *If a general monic quintic in its various forms is in general irreducible in respect to the other lower degree polynomials then it is relatively prime*
Proposition 4: *When a polynomial has no rational roots in a field of rational numbers, then the field of rational numbers can be extended a larger field that can*
I will seek, by different approaches the solvable form of the general quintic.
APPROACH 1
Consider the general quintic equation:
$x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0$ −−−−−−−−−−−−−−−−−−−−−−−−−− 1
If we permitted a functional form the coefficients such that:
$a_4 = k_1x$ −−−−−−−−−−−−−−−−− 2
$a_3 = k_2x^2$ −−−−−−−−−− 3
$a_2 = k_3x^3$ −−−−−−−−−−− 4
$a_1 = k_4x^4$ −−−−−−−−−−−− 5
$a_0 = k_5x^5$ −−−−−−−−−−−− 6
And: k_1, k_2, k_3, k_4, k_5 *are in the field of rational numbers.*
We can substitute equations 2 *to* 6 *into* 1 *to obtain the equation*:
$k_1 + k_2 + k_3 + k_4 + k_5 + 1 = 0$
Or $k_5 = -(k_1 + k_2 + k_3 + k_4 + 1)$ −−−−−−−−−−−−−−−−−− 7

We can substitute equations 7 *to* 6

$a_0 = k_5x^5 = -(k_1 + k_2 + k_3 + k_4 + 1)x^5$ −−−−−−−−−−−− 6

$$x^5 = \frac{-a_0}{(k_1 + k_2 + k_3 + k_4 + 1)} \text{ −−−−−−−−−−−− } 6$$

By 7 *and* 6,

$$x = \sqrt[5]{\frac{-a_0}{k_1 + k_2 + k_3 + k_4 + 1}} \text{ −−−−−−−−−−−−−−−−−−−− } 8$$

Substituting 8 *into* 2 *to* 5:

$$a_4 = k_1\sqrt[5]{\frac{-a_0}{k_1 + k_2 + k_3 + k_4 + 1}} \text{ −−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−− } 9$$

$$a_3 = k_2\sqrt[5]{\left(\frac{-a_0}{k_1+k_2+k_3+k_4+1}\right)^2} -------------------- 10$$

$$a_2 = k_3\sqrt[5]{\left(\frac{-a_0}{k_1+k_2+k_3+k_4+1}\right)^3} -------------------- 11$$

$$a_1 = k_4\sqrt[5]{\left(\frac{-a_0}{k_1+k_2+k_3+k_4+1}\right)^4} ------------------ 12$$

When the general quintic equation is written in the form:

$$x^5 + \left(k_1\sqrt[5]{\frac{-a_0}{k_1+k_2+k_3+k_4+1}}\right)x^4 + \left(k_2\sqrt[5]{\left(\frac{-a_0}{k_1+k_2+k_3+k_4+1}\right)^2}\right)x^3 + \left(k_3\sqrt[5]{\left(\frac{-a_0}{k_1+k_2+k_3+k_4+1}\right)^3}\right)x^2 + \left(k_4\sqrt[5]{\left(\frac{-a_0}{k_1+k_2+k_3+k_4+1}\right)^4}\right)x + a_0 = 0 ------ 13,$$

One of its roots is:

$$x = \sqrt[5]{\frac{-a_0}{k_1+k_2+k_3+k_4+1}} ------------------ 8$$

$$x_1 = \sqrt[5]{\frac{-a_0}{k_1+k_2+k_3+k_4+1}} -------------------- 14$$

Coefficients of quintic equation 14 *are in radical form while those of equation* 1 *are not*

If the coefficients of the quintic (13)*are assumed to be real rational numbers then*

$$\frac{-a_0}{k_1+k_2+k_3+k_4+1} = x^5 = q^5 --------- 15,$$

$$x = q = \sqrt[5]{\frac{-a_0}{k_1+k_2+k_3+k_4+1}} ------------------ 15,$$

q is a rational number

Substituting 15 *into equation* 13:

$$x^5 + (k_1q)x^4 + (k_2q^2)x^3 + (k_3q^3)x^2 + (k_4q^4)x - (k_1+k_2+k_3+k_4+1)q^5 = 0 -------- 16$$

$$\frac{x^5 + (k_1q)x^4 + (k_2q^2)x^3 + (k_3q^3)x^2 + (k_4q^4)x - (k_1+k_2+k_3+k_4+1)q^5}{x-q} = (x^4 + ax^3 + bx^2 + cx + d) = 0 -------- 16$$

Equation 16 can be factorized into the form:

$$x^5 + (k_1 q)x^4 + (k_2 q^2)x^3 + (k_3 q^3)x^2 + (k_4 q^4)x - (k_1 + k_2 + k_3 + k_4 + 1)q^5 = (x - q)(x^4 + ax^3 + bx^2 + cx + d) = 0 \quad ------------ \; 17$$

Where:

$$a = q(k_1 + 1) \quad -------------------- \; 18$$

$$b = q^2(k_2 + k_1 + 1) \quad -------------------- \; 19$$

$$c = q^3(k_3 + k_2 + k_1 + 1) \quad --------------- \; 20$$

$$d = q^4(k_4 + k_3 + k_2 + k_1 + 1) \quad ------------- \; 21$$

Either $x - q = 0 \Rightarrow x = q$

Or $(x^4 + ax^3 + bx^2 + cx + d) = 0$

By equation 16 given rational numbers $k_1, k_2 k_3, k_4$ and q one should be able to determine the roots of the quintic. By equation 16 if these parameters are in the fie

Quintic equation 16 is a composite form of equation 13 (a composite polynomial is reducible to lower degree factors). The connections between the coefficients o

$a_4 = k_1 q$

$a_3 = k_2 q^2$

$a_2 = k_3 q^3$

$a_1 = k_4 q^4$

$a_0 = -(k_1 + k_2 + k_3 + k_4 + 1)q^5$

Example

If we take $k_1 = k_2 = k_3 = 0, k_4 = 1$ and $q = 1$ we obtain the quintic $x^5 + x - 2 = 0$

With root $x_1 = q = 1$

The method above integrates one of the roots of the quintic as one of the constituent parameters of the coefficients.

By the above reasoning it can be shown that if the Bring − Jerrard quintic is written in the form:

$$x^5 + bx - (n+1)\left(\frac{b}{n}\right)^{\frac{5}{4}} = 0 \quad ------------------- \; 22$$

Then one of the roots is given by:

$$x_1 = \sqrt[4]{\frac{b}{n}} \quad ----------------- \; 23$$

Note here, when the Bring − Jerrard quintic is written in the solvable form 22, one of the roots is in the form,

$x_1 = \sqrt[4]{l}$. It will be established, the form the first root of the equation takes depends on the form of the solvable form chosen.

If $b = 81$ and $n = 16$ then the resulting quintic equation is:

$$x^5 + 81x - \frac{4131}{32} = 0 \quad ---------- \; 24$$

One of the roots of this quintic equation is $x_1 = \frac{3}{2}$

For the form 22 to accommodate only rational coefficient I will make

$$\frac{b}{n} = p^4 - - - - - - -5$$

Where p is a rational number, in which case equation 23 can be written in the form:

$$x^5 + np^4x - (n+1)p^5 = 0 \ - - - - - - - - - - - - - - - - - - 24$$

In which case the first root equation 24 is given by:

$$x_1 = p \ - - - - - - - - - - \ 25$$

Equation 24 suggests that Bring − Jerrard quintics with a common root say, $\frac{12}{13}$ will take the form:

$$x^5 + \frac{20736}{28561}nx - \frac{248832}{371293}(1+n) = 0$$

Equation forms:

$$x^5 + \frac{2401}{1296}nx - \frac{16807}{7776}(1+n) = 0$$, will have a common root, $x_1 = \frac{7}{6}$

In the above set of Bring − Jerrard quintics we can identify a quintic like:

$$x^5 - \frac{59023783}{21781872}x + 1 = 0$$

Such an equation has a root $x_1 = \frac{7}{6}$

Equation 24 permits one to introduce a Bring − Jerrard quintic triplet co

− ordinates, (b, c, x_1) where b is the x coefficient of the quintic, c is the constant term of the quintic and x_1 is one of the roots of the quintic . By equation 24 the tri

$+ 1)p^5, p)$. These triplets will permit a construction of R^3 space of the Bring

− Jerrard quintic. The selection of specific values of the parameters constituting the triplets will permit the identification of the specific quintic. Suppose in this

$= 1$, then the Bring − Jerrard triplets of the subspace would be $(n, -(n+1), 1)$. This would be a subspace containing quintics having a root, $x_1 = 1$.

The above simple analysis has helped to convert general and Bring − Jerrard quintics to solvable forms.

RADICAL SOLUTION OF THE BRING − JERRARD QUINTIC – ALTERNATIVE APPROACH 2

Consider the Bring − Jerrard quintic equation :

$$x^5 + bx + c = 0 \ - - - - - - - - - - - - - - 1$$

One way of obtaining its solution is to consider a functional form of the coefficient b such that

$$b = (q^5 - 1)x^4 - - - - - - - - - - - - - - - - - - - 2$$

Where q is in the field of rational numbers.

Then a substitution of 2 into 1 would yield a real root of x given by

$$x_1 = \left(\frac{-c}{q}\right)^{\frac{1}{5}} - 3$$

$Substituting\ 3\ into\ 2\ we\ would\ obtain\ the\ relation:$

$$b = f(q,c) = (q^5 - 1)\frac{(-c)^{\frac{4}{5}}}{q^4} - - - - - - - - - - - - - - - - - - - 4$$

$This\ means\ that\ is\ the\ Bring\ Jerrard\ quintic\ is\ written\ in\ the\ form:$

$$x^5 + (q^5 - 1)\frac{(-c)^{\frac{4}{5}}}{q^4}x + c = 0 - (5), then\ one\ of\ the\ real\ roots\ of\ the\ quintic\ equation\ is\ given\ by:$$

$$x_1 = \left(\frac{-c}{q}\right)^{\frac{1}{5}}$$

$If\ we\ take\ c = r^5 then\ equation\ 5\ takes\ the\ form:$

$$x^5 + (q^5 - 1)\frac{(-r^5)^{\frac{4}{5}}}{q^4}x + c = x^5 + (q^5 - 1)\frac{-r^4}{q^4}x + c = 0\ - - - - - - - - - -\ 6$$

$$In\ which\ case\ x_1 = \left(\frac{-c}{q}\right)^{\frac{1}{5}} = \left(\frac{-r^5}{q}\right)^{\frac{1}{5}} = \frac{-r}{q^{\frac{1}{5}}}$$

$If\ we\ take\ r\ = 1\ and\ q\ = 2\ we\ obtain\ the\ quintic\ equation:$

$$x^5 + (q^5 - 1)\frac{-r^4}{q^4}x + c = x^5 + \frac{31}{16}x + 1 = 0$$

$$With\ a\ root: x_1 = \frac{-r}{q^{\frac{1}{5}}} = -\frac{1}{2}$$

$RADICAL\ SOLUTION\ OF\ THE\ BRING - \ JERRARD\ QUINTIC - \ ALTERNATIVE\ APPROACH\ 3$

$Consider\ again\ the\ general\ quintic\ equation:$

$$x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0\ - 1$$

$If\ we\ take$

$$a_2 = l_1x\ - - - - - - - - - - - - - - - - - - - 2$$

$$a_1 = l_2x^2 -\ 3$$

$$a_0 = l_3x^3 - - - - - - - - - - - - - - - - - - -\ 4\ and\ substituted\ into\ equation\ 1\ we\ would\ get\ the\ value:$$

$$x_{1,2} = \frac{-a_4 \pm \sqrt{a_4^2 - 4(l_1 + l_2 + l_3 + a_3)}}{2} - 5$$

$If\ we\ wrote\ equation\ 1\ in\ the\ form:$

$$x^5 + a_4x^4 + a_3x^3 + l_1\left(\frac{-a_4 + \sqrt{a_4^2 - 4(l_1 + l_2 + l_3 + a_3)}}{2}\right)x^2 + l_2\left(\frac{-a_4 + \sqrt{a_4^2 - 4(l_1 + l_2 + l_3 + a_3)}}{2}\right)^2 x + l_3\left(\frac{-a_4 + \sqrt{a_4^2 - 4(l_1 + l_2 + l_3 + a_3)}}{2}\right)^3 = 0\ - - - - - - - - - - -\ 6$$

$In\ the\ form\ 6\ above\ the\ quintic\ equation\ has\ root:$

$$x_1 = \frac{-a_4 + \sqrt{a_4^2 - 4(l_1 + l_2 + l_3 + a_3)}}{2} ----------------- 7$$

$If\ we\ let\ a_4 = a_3 = l_1 = 0$

$We\ obtain\ the\ Bring\ Jerrard\ quintic\ form$

$$x^5 + l_2\left(\sqrt{-(l_2 + l_3)}\right)^2 x + l_3\left(\sqrt{-(l_2 + l_3)}\right)^3 = 0 -------------------- 8a$$

$Of\ the\ general\ form: x^5 + bx + c = 0 -------------- 8b$

$$x_1 = \sqrt{-(l_2 + l_3)} ---------------- 9$$

$If\ we\ selected\ l_2 and\ l_3 such\ that:$

$$l_2 + l_3 = -1 ----- 10$$

$$x_1 = \frac{+\sqrt{-4(l_2 + l_3)}}{2} = 1$$

$$b = l_2 t^2 = l_2 ------------ 12$$

$$c = l_3 t^3 = -(1 + l_2)$$

$All\ Bring - Jerrard\ quintics\ of\ the\ form:$

$x^5 + l_2 x - (1 + l_2) = 0\ , have\ a\ root\ x_1 = 1$

$If\ we\ select\ l_2 + l_3 = -p$

$Then\ the\ Bring - Jerrard\ quintic\ generated\ is\ of\ the\ form:$

$$x^5 + pl_2 x - p^{\frac{3}{2}}(p + l_2) = 0 ---------- 13,$$

$And\ has\ one\ of\ its\ roots\ as\ x_1 = +\sqrt{p} -------------- 14$

$In\ equation\ 13\ if\ p = 4\ and\ l_2 = 1\ then\ we\ would\ from\ these\ values\ obtain\ the\ quintic\ equation\ x^5 + 4x - 40 = 0\ with\ of\ the\ roots\ given\ by\ x_1 = +\sqrt{p} = 2$

$A\ solvable\ form\ of\ the\ quintic\ can\ be\ selected\ in\ which\ the\ first\ root\ is\ of\ the\ form\ x_1 = \sqrt[3]{l}$

$To\ have\ rational\ coefficients\ I\ take\ the\ substitution$

$$p = t^2 -------------------- 15$$

$In\ which\ case:$

$$x^5 + t^2 l_2 x - t^3(t^2 + l_2) = 0 -------------- 16$$

$The\ first\ root\ of\ the\ above\ quintic\ is:$

$$x_1 = t ------------- 17$$

$METHODS\ OF\ SOLVING\ SEXTIC\ EQUATIONS$

$Approach\ 1 - Radical\ solution\ of\ the\ sextic\ equation$

$The\ general\ sextic\ equation\ can\ be\ reduced\ to\ one\ with\ three\ parameters\ by\ the\ Bring - Jerrard\ transformation\ and\ take\ a\ form$

$$x^6 + bx^2 + cx + d = 0 ----------1$$

$Using\ the\ method\ used\ in\ alternative\ approach\ 3\ above\ we\ can\ express\ the\ three\ coefficients\ in\ terms\ of\ three\ rational\ l_1, l_2, l_3 such\ that\ l_1 + l_2 + l_3 = -p\ ---\ (2) and\ write\ the\ sextic\ equation\ 1\ above\ the\ general\ solvable\ form\ below.$

$$x^6 + l_1 p x^2 + l_2 p^{\frac{3}{2}} x + l_3 p^2 = 0 \quad ----------- 3$$

$In\ the\ above\ form\ x_1 = \sqrt{p} --------- 4$

$If\ we\ take: l_1 = -2, l_2 = 0\ l_3 = -2\ then\ p = 4\ (by\ 2). We\ would\ then\ obtain\ the\ sextic\ equation\ x^6 - 8x^2 - 32 = 0\ with\ one\ of\ the\ roots\ given\ by: x_1 = +\sqrt{p} = 2$

$Approach\ 2$

$Consider\ the\ general\ sixth\ degree\ polynomial,$

$$x^6 + a_5 x^5 + a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0 \quad ---------------------------- 1$$

$In\ my\ published\ paper\ entitled$ “$A\ Calculator\ for\ Polynomial\ Equations\ of\ Degree\ Five\ and\ above$” [1], $I\ proposed\ a\ general\ method\ of\ factorizing\ a\ polynomial\ o$

$Suppose\ the\ equation\ 1\ above\ is\ allowed\ to\ factorize\ to\ the\ form:$

$$(x - r_1)(x - r_2)(x^4 + c_3 x^3 + c_2 x^2 + c_1 x + c_0) = 0 \quad ----------------------- 2;\ where:$$

$$a_0 = r_1 r_2 c_0 --------- 3$$

$$a_1 = c_1 r_1 r_2 - c_0(r_1 + r_2) ---------------- 4$$

$$a_2 = c_2 r_1 r_2 - c_1(r_1 + r_2) + c_0 ----------------- 5$$

$$a_3 = c_3 r_1 r_2 - c_2(r_1 + r_2) + c_1 ----------------- 6$$

$$a_4 = r_1 r_2 - c_3(r_1 + r_2) + c_2 -------------------- 7$$

$$a_5 = c_3 - r_1 - r_2 ----------------- 8$$

$The\ general\ sextic\ equation\ can\ be\ written\ in\ the\ form\ below\ for\ easy\ factorization$

$$x^6 + (c_3 - r_1 - r_2)x^5 + (r_1 r_2 - c_3(r_1 + r_2) + c_2)x^4 + (c_3 r_1 r_2 - c_2(r_1 + r_2) + c_1)x^3 + (c_2 r_1 r_2 - c_1(r_1 + r_2) + c_0)x^2 + \left(c_1 r_1 r_2 - c_0(r_1 + r_2)\right)x + r_1 r_2 c_0 = 0 \quad --------------------- 9$$

$The\ factorized\ form\ is\ given\ in\ equation\ 2.$

$The\ coefficients\ of\ the\ quintic\ are\ replaced\ by\ rational\ r_1, r_2, c_0, c_1, c_2 and\ c_3$

$If\ we\ chose\ c_1 = c_2 = c_3 = 0, r_1 = r_2 = 1\ and\ c_0 = 1\ and\ substituted\ into\ equation\ 9\ we\ would\ obtain\ the\ quintic\ equation$

$$x^6 - 2x^5 + x^4 + x^2 - 2x + 1 = 0$$

$With\ double\ roots\ x_{1,2} = r_1 = r_2 = 1. The\ other\ roots\ are\ obtained\ by\ solving\ the\ auxiliary\ quartic\ equation.$

$Equation\ 9\ can\ be\ written\ in\ the\ form:$

$$x^6 + (c_3 - t)x^5 + (p - c_3 t + c_2)x^4 + (c_3 p - c_2 t + c_1)x^3 + (c_2 p - c_1 t + c_0)x^2 + (c_1 p - c_0 t)x + p c_0 = 0 \quad -------- 10$$

$Where:$

$$r_1 r_2 = p$$

$$r_1 + r_2 = t$$

$In\ the\ above\ form: x_{1,2} = \dfrac{t \pm \sqrt{t^2 - 4p}}{2}$

$This\ method\ can\ be\ extended\ to\ higher\ degree\ polynomials.$

METHODS OF SOLVING SEPTIC EQUATIONS

Consider the general seventh degree polynomial,

$x^7 + a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0$ ——————————————————————— 1

Suppose the equation 1 *above is allowed to factorize to the form*:

$(x - r_1)(x - r_2)(x - r_3)(x^4 + c_3x^3 + c_2x^2 + c_1x + c_0) = 0$ ——————————————3

where:

$a_0 = -r_1r_2r_3c_0$ ————————— 3

$a_1 = -c_1r_1r_2r_3 + c_0(r_1r_2 + r_1r_3 + r_2r_3)$ ——————————————— 4

$a_2 = -c_2r_1r_2r_3 + c_1(r_1r_2 + r_1r_3 + r_2r_3) - c_0(r_1+r_2 + r_3)$ ———————————————— 5

$a_3 = -c_3r_1r_2r_3 + c_2(r_1r_2 + r_1r_3 + r_2r_3) - c_1(r_1+r_2 + r_3) + c_0$ ———————————————— 6

$a_4 = -r_1r_2r_3 + c_3(r_1r_2 + r_1r_3 + r_2r_3) - c_2(r_1+r_2 + r_3) + c_1$ ———————————————————7

$a_5 = r_1r_2 + r_1r_3 + r_2r_3 - c_3(r_1+r_2 + r_3) + c_2$ ———————————————— 8

$a_6 = -(r_1+r_2 + r_3) + c_3$ ———————————————————9

If we take $r_1+r_2 + r_3 = p$ ————————————— 10

$r_1r_2 + r_1r_3 + r_2r_3 = q$ ———————————————— 11

$r_1r_2r_3 = r$ ————————————— 12

where:

$a_0 = (-rc_0)$ ————————— 3

$a_1 = \left(-c_1r + c_0(q)\right)$ ——————————————— 4

$a_2 = \left(-c_2r + c_1(q) - c_0(p)\right)$ ———————————————— 5

$a_3 = (-c_3r + c_2(q) - c_1(p) + c_0)$ ———————————————— 6

$a_4 = (-r + c_3(q) - c_2(p) + c_1)$ ———————————————————7

$a_5 = (q - c_3(p) + c_2)$ ———————————————— 8

$a_6 = (-(p) + c_3)$ ———————————————————9

Equation 3 *takes the form*:

$$x^7 + (-(p) + c_3)x^6 + (q - c_3(p) + c_2)x^5 + (-r + c_3(q) - c_2(p) + c_1)x^4 + (-c_3r + c_2(q) - c_1(p) + c_0)x^3 + \left(-c_2r + c_1(q) - c_0(p)\right)x^2 + \left(-c_1r + c_0(q)\right)x + (-rc_0) =$$

$(x - r_1)(x - r_2)(x - r_3)(x^4 + c_3x^3 + c_2x^2 + c_1x + c_0) = 0$ ——————————— 13

From equations 10 – 12 *we obtain the equation*

$$r_1^3 - pr_1^2 + qr_1 - r = 0 \; ------------ \; 14$$

Three of the roots of equation 14 *are also the roots of the septic equation* 13

One of the roots of equation 14 *is*:

$$r_{11} = \sqrt[3]{\left(\frac{1}{2}\left(\frac{2p^3}{27} + r - \frac{pq}{3}\right) - \sqrt{\frac{1}{4}\left(\frac{2p^3}{27} + r - \frac{pq}{3}\right)^2 + \left(q - \frac{p^2}{3}\right)}\right)^3} - \frac{p}{3} -------------- \; 15$$

The other three roots of equation 14 *are obtained by solving the auxiliary quadratic equation*:

$$r_1^2 + (r_{11} - p)r_1 + \frac{r}{r_{11}} = 0$$

$$r_{12,13} = \frac{p - r_{11} \pm \sqrt{(p - r_{11})^2 - \frac{4r}{r_{11}}}}{2} ---------------- \; 16$$

REFERENCES

Buya S. B. (2013). *A calculator for polynomial equations of degree five and above. Open science repository mathematics, Online* (*open access*), *e*23050424. *doi*: $\frac{10}{ope}$

Buya S. B. (2013). *Disproof Of Abel's impossibility theorem. Open science repository mathematics, Online* (*open access*), 23050460. *doi*; $\frac{10.7392}{openaccess}$. 230504

E. S. Bring, Quart. J. Math., 6 (1864).

E. S. Bring, Meletemata quaedam mathematematica circa transformationem aequationum algebraicarum, Lund, 1786.

J. C. Glashan. Notes on the quintic. Amer. J. Math., 8: 178– 179, 1885.

E T Motlotle – 2011. *The Bring* − *Jerrard quintic equation, its solutions and a formula for the universal gravitational constant* – *umkn* − *dsp*01. *unisa. ac. za*