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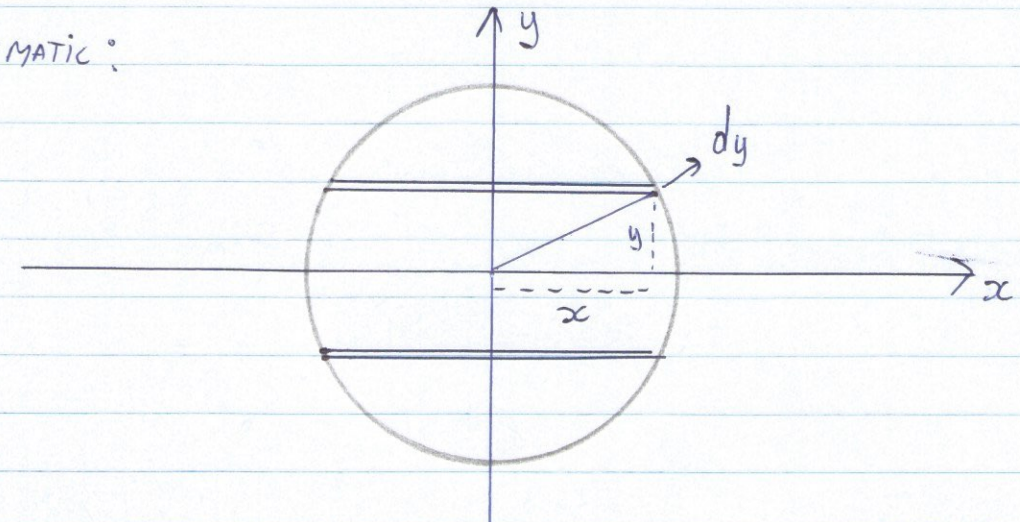
WE TAKE AN ELEMENTARY PARTICLE AS A SPHERE/GLOBE WITH A RADIUS r. WE LOOK AT IT FROM THE CENTER OF THE SPHERE, IN A SPECIAL RELATIVISTIC WAY. THE SPHERE HAS A DENSITY OF ρ_0. THE REST MASS OF THE ELEMENTARY PARTICLE IS THEN:

$$m_0 = \frac{4}{3}\pi . \rho_0 . r^3$$

THE SPHERE ROTATES AROUND THE X-AXIS. WE NOW LOOK AT A CILINDER SHELL PARALEL TO THE X-AXIS WHICH ROTATES AT A CONSTANT VELOCITY V: $V = \omega . y$ IN WHICH ω IS THE ANGULAR VELOCITY.

SCHEMATIC:



SPERE EQUATION (CIRCLE): $x^2 + y^2 = r^2 \Rightarrow x = \sqrt{r^2 - y^2}$

VOLUME OF THIS VERY THIN CILINDER SHELL IS: $dV = 2\pi y . 2x . dy = 4\pi x . y . dy$
$$= 4\pi \sqrt{r^2 - y^2} . y \, dy$$

WE NOW CALCULATE THE ROTATIONAL ENERGY OF THIS CILINDER SHELL THAT CROSSES THE SPHERE:

$$dE = c^2 . dm = c^2 . \rho . dV = 4\pi \rho . \sqrt{r^2 - y^2} . y \, dy$$

WITH $\rho = \rho_0 . \frac{c}{\sqrt{c^2 - V^2}} = \rho_0 . \frac{c}{\sqrt{c^2 - \omega^2 r^2}}$ dE BECOMES:

$$dE = 4\pi \rho_0 . c^3 \sqrt{\frac{r^2 - y^2}{c^2 - \omega^2 y^2}} . y . dy$$

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THE TOTAL KINETIC ROTATIONAL ENERGY BECOMES THEN:

$$E = \int_0^r dE = \int_0^r 4\pi \rho_0 c^3 \sqrt{\frac{r^2 - y^2}{c^2 - \omega^2 y^2}} \, . \, y \, dy$$

WE CAN SIMPLIFY ALREADY IN THE FOLLOWING WAY:

$$E = \int_0^r \frac{4\pi}{2} \rho_0 c^3 \sqrt{\frac{r^2 - y^2}{c^2 - \omega^2 y^2}} \, dy^2 \; ;$$ WE NOW CHANGE VARIABLES: $y^2 = u$

$$E = \int_{y=0}^{y=R} 2\pi \rho_0 c^3 \sqrt{\frac{r^2 - u}{c^2 - \omega^2 u}} \, . \, du \; ;$$ WE CONTINUE TO SIMPLIFY: $r^2 - u = m$

$u = r^2 - m$

$$\Rightarrow E = \int_{y=0}^{y=R} -2\pi \rho_0 c^3 \sqrt{\frac{m}{(c^2 - \omega^2 R^2) + \omega^2 m}} \, . \, dm = \int_{y=0}^{y=r} -2\pi \rho_0 \frac{c^3}{\omega} . \sqrt{\frac{m}{\left(\frac{c^2}{\omega^2} - r^2\right) + m}} \, . \, dm$$

UPPER LIMIT: $y_2 = r \Rightarrow u_2 = r^2 \Rightarrow m_2 = 0$

LOWER LIMIT: $y_1 = 0 \Rightarrow u_1 = 0 \Rightarrow m_1 = R^2$

AND: $\frac{c^2}{\omega^2} - r^2 = a$ WE GET:

$$E = \int_{R^2}^{0} -2\pi \rho_0 \frac{c^3}{\omega} \sqrt{\frac{m}{a + m}} \, . \, dm$$ WITH THE SOLUTION:

$$E_{ROT} = -2\pi \rho_0 \frac{c^3}{\omega} \left[\sqrt{m} . \sqrt{m + a} - a \ln\left(\sqrt{m} + \sqrt{m + a}\right) \right]_{R^2}^{0}$$

THIS RESULTS IN: $E_{ROT} = \pi \rho_0 \frac{c^3}{\omega^3} \left(2 \omega r c - (c^2 - \omega^2 r^2) \ln\left(\frac{c + \omega r}{c - \omega r}\right) \right)$