A Smooth Exit from Eternal Inflation?

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Abstract

The usual theory of inflation breaks down in eternal inflation. We derive a dual description of eternal inflation in terms of a deformed CFT located at the threshold of eternal inflation. The partition function gives the amplitude of different geometries of the threshold surface in the Hartle-Hawking state. Its local and global behavior in dual toy models shows that the amplitude is low for surfaces which are not nearly conformal to the round three-sphere and essentially zero for surfaces with negative curvature. Based on this we conjecture that the exit from eternal inflation does not produce an infinite fractal-like multiverse, but is finite and reasonably smooth.

I. INTRODUCTION

Eternal inflation [1] is a near de Sitter regime deep into the phase of inflation in which the quantum fluctuations in the energy density of the inflaton are large. In the usual account of eternal inflation the quantum diffusion dynamics of the fluctuations is modeled as stochastic effects around a classical slow roll background. Since the stochastic effects dominate the classical slow roll it is argued eternal inflation produces universes that are globally highly irregular, with exceedingly large or infinite constant density surfaces [2–5].

However this account is questionable, because the dynamics of eternal inflation wipes out the separation into classical backgrounds and quantum fluctuations that is assumed. We therefore put forward a different model of eternal inflation based on gauge-gravity duality. A reliable theory of eternal inflation is important to sharpen the predictions of slow roll inflation. This is because the dynamics of eternal inflation specifies the prior over the zero modes, or slow roll backgrounds, in the theory which in turn determines its predictions for the precise spectral properties of CMB fluctuations on observable scales.

We work in the no-boundary quantum state [6]. This gives the ground state and is heavily biased towards universes with a low amount of inflation [7]. However we do not observe the entire universe. Instead our observations are limited to a small patch mostly along part of our past light cone. Probabilities for local observations in the no-boundary state are weighted by the volume of a surface Σ_f of constant measured density, to account for the different possible locations of our past light cone [8]. This transforms the probability distribution for the amount of inflation and leads to the prediction that our universe emerged from a regime of eternal inflation [8, 9]. Thus we must understand eternal inflation in order to understand the observational implications of the no-boundary wave function.

The standard saddle point approximation of the no-boundary wave function breaks down in eternal inflation. However, gauge-gravity duality or dS/CFT [10–12] enables an alternative form of the wave function on a surface Σ_f in the large three-volume limit in which the wave function is specified in terms of the partition function of certain deformations of a Euclidean CFT defined on Σ_f. Euclidean AdS/CFT generalized to complex relevant deformations implies an approximate realisation of this [13–18]. This follows from the observation [17] that *all* no-boundary saddle points in low energy gravity theories with a positive scalar potential V admit a geometric representation in which their weighting is fully specified by

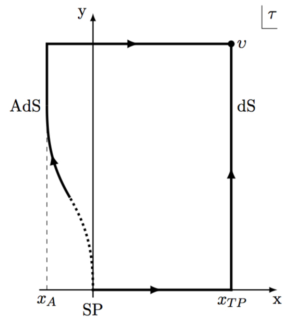

FIG. 1: Two representations in the complex time-plane of the same no-boundary saddle point associated with an inflationary universe. The saddle point action includes an integral over time τ from the no-boundary origin or South Pole (SP) to its endpoint υ on Σ_f. Different contours for this give different geometric representations of the saddle point, each giving the same amplitude for the final real configuration $(h_{ij}(\vec{x}), \phi(\vec{x}))$ on Σ_f. The interior saddle point geometry along the nearly vertical contour going upwards from the SP consists of a regular, Euclidean, locally AdS domain wall with a complex scalar profile. Its regularized action specifies the tree-level probability in the no-boundary state of the associated inflationary, asymptotically de Sitter history. Euclidean AdS/CFT relates this to the partition function of a dual field theory yielding (1.1).

an interior, locally AdS, domain wall region governed by an effective negative scalar potential $-V$. We illustrate this in Fig. 1. Quantum cosmology thus lends support to the view that Euclidean AdS/CFT and dS/CFT are two real domains of a single complexified theory [10, 13, 19–21]. In the large three-volume limit this has led to the following proposal for a holographic form of the semiclassical no-boundary wave function [17],

$$\Psi_{NB}[h_{ij}, \phi] = Z^{-1}_{QFT}[\tilde{h}_{ij}, \tilde{\alpha}] \exp(iS_{st}[h_{ij}, \phi]/\hbar) \ . \tag{1.1}$$

Here the sources $(\tilde{h}_{ij}, \tilde{\alpha})$ are conformally related to the argument (h_{ij}, ϕ) of the wave function, S_{st} are the usual surface terms, and Z_{QFT} in this form of dS/CFT are partition functions of deformations of Euclidean AdS/CFT duals. The boundary metric $\tilde{h}_{ij}$ stands for background *and* fluctuations.

The holographic form (1.1) has led to a fruitful and promising application of holographic techniques to early universe cosmology (see e.g. [14, 22–26]). No field theories have been identified that correspond to top-down models of realistic cosmologies where inflation transitions to a decelerating phase. However it turns out that many of the known AdS/CFT duals are ideally suited to study eternal inflation from a holographic viewpoint. This is because supergravity theories in AdS_4 typically contain scalars of mass $m^2 = -2l^2_{AdS}$ and with a negative exponential potential for large ϕ. This gives rise to eternal inflation in the dS domain of the theory governed effectively by $-V$. In fact the Breitenlohner-Freedman bound in AdS corresponds precisely to the condition for slow roll eternal inflation in dS.

In this paper we consider toy-model cosmologies of this kind in which a single bulk scalar drives slow roll eternal inflation. Conventional wisdom based on semiclassical gravity asserts that surfaces of constant scalar field become highly irregular on the largest scales. We study this from a holographic perspective with the dual defined on a global constant density surface Σ_f in the far future. The holographic form (1.1) of the wave function replaces the bulk scalar by a source $\tilde{\alpha}$ that turns on a low dimension scalar operator in the dual. The dependence of the partition function on the geometry h_{ij} of the future conformal boundary in the presence of a constant source $\tilde{\alpha} \neq 0$ then specifies a holographic measure on the global structure of constant density surfaces in eternal inflation in the large volume regime. In this dual framework, the separate bulk background and fluctuation degrees of freedom in traditional approaches to eternal inflation are replaced by field theory degrees of freedom on a future 'end-of-the-world' brane. As such the field theory automatically includes the backreaction effect of the fluctuations on the classical background[1].

The F–theorem and its extensions imply that the holographic amplitude of the round S^3 is a local maximum of the distribution, in contrast with expectations based on semiclassical gravity. Here we compute the global holographic probability distribution for small and large anisotropic deformations (squashings) of the future boundary in dual vector models. We find that the amplitude of surfaces with conformal structures far from the round one is exponentially small. Finally we put forward a general argument that indicates that the amplitude is zero for all highly deformed conformal boundaries with a negative Yamabe invariant. This raises doubt about the widespread idea that eternal inflation produces a highly irregular universe with a mosaic structure of bubble like patches separated by inflationary domains.

[1] In a complete model of cosmology Σ_f should be thought of as a surface at the threshold of eternal inflation. However it is beyond the current state of holographic cosmology to study the transition from eternal inflation to classical cosmology.

II. A HOLOGRAPHIC MEASURE ON ETERNAL INFLATION

As an example of a bulk gravity model that leads to eternal inflation in the dS domain of the theory we consider the well known consistent truncation of M-theory on $AdS_4 \times S^7$ down to gravity coupled to a single scalar ϕ with potential

$$V(\phi) = -2 - \cosh(\sqrt{2}\phi)\,, \tag{2.1}$$

in units where $\Lambda = -3$ and hence $l^2_{AdS} = 1$. The scalar has mass $m^2 = -2$. Therefore in the large three-volume regime it behaves as

$$\phi(\vec{x}, r) = \alpha(\vec{x})e^{-r} + \beta(\vec{x})e^{-2r} + \cdots \tag{2.2}$$

where r is the overall radial coordinate in Euclidean AdS, with scale factor e^r. The Fefferman-Graham expansion implies that in terms of the variable r the asymptotically (Lorentzian) dS domain of the theory is to be found along the vertical line $\tau = r + i\pi/2$ in the complex τ-plane [17]. This is illustrated in Fig. 1 where r changes from real to imaginary values along the horizontal branch of the AdS contour from x_A to x_{TP}. This also means that in the dS domain the potential (2.1) in the original Euclidean action of the AdS theory acts as a positive effective potential

$$\tilde{V}(\phi) = -V = 2 + \cosh(\sqrt{2}\phi)\ . \tag{2.3}$$

This is a potential for which the eternal inflation condition $\epsilon \leq \tilde{V}$ holds, with $\epsilon \equiv \tilde{V}^2_{,\phi}/\tilde{V}^2$, for field values in the neighbourhood of its minimum. This close connection between AdS supergravity truncations and eternal inflation stems from the fact that the Breitenlohner-Freedman stability bound on the mass of scalars in AdS corresponds precisely to the condition for eternal inflation in the de Sitter domain of the theory.

The variance of the wave function of inhomogeneous fluctuation modes is of order $\sim \tilde{V}/\epsilon$ evaluated at horizon crossing. Hence the fluctuation wave function is broadly distributed [5]. This is the hallmark of eternal inflation in quantum cosmology. It implies the universe's evolution is governed by the quantum diffusion dynamics of the fluctuations and their back-reaction on the geometry rather than the classical slow roll [2–5]. As a consequence it is usually argued the total wave function spreads out and loses its semiclassical WKB form.

The argument (h_{ij}, ϕ) of the wave function evaluated at υ in Fig. 1 is real. This means that in saddle points associated with inflationary universes, the scalar field must become real along the vertical dS line in the τ-plane. The expansion (2.2) shows this requires its leading coefficient α to be imaginary, which in turn means that the scalar profile is complex along the entire interior AdS domain wall part of the saddle points. But the bulk scalar sources a deformation by an operator $\mathcal{O}$ of dimension one with coupling α in the dual ABJM theory. Hence the holographic measure on the ensemble of eternally inflating universes in this model is specified by its partition function on deformed three-spheres in the presence of an imaginary mass deformation. We first recall the general behavior of partition functions for small perturbations away from the round S^3 and then we turn to large deformations in a specific toy model.

A. Local measure: perturbations around S^3

Locally around the round sphere, the F–theorem and its extension to spin-2 deformations provide a general argument that the round sphere is a local minimum of the partition function. The F–theorem for three-dimensional CFTs [27, 28] states that the free energy of a CFT on S^3 decreases along an RG flow triggered by a relevant deformation. A similar result was recently proved for metric perturbations of the conformal S^3 background [29, 30]. The coupling of the energy-momentum tensor of the CFT to the curved background metric triggers a spin-2 deformation. The fact that the free energy is a local maximum for the round sphere is essentially equivalent to the positive definiteness of the stress tensor two-point function. Applied to the holographic no-boundary wave function (1.1) these results imply that the pure de Sitter history in the bulk is a local maximum of the holographic probability distribution, in contrast with expectations based on semiclassical bulk gravity in eternal inflation.

B. Global measure: squashed three-spheres

We now turn to large deformations. Our bulk model is a consistent truncation of M-theory compactified on $AdS_4 \times S^7$. Therefore the dual is the ABJM SCFT, and we are faced with the problem of evaluating the partition function of supersymmetry breaking deformations of this theory. We do not attempt this here. Instead we focus on a simplified model of this setup where we consider an $O(N)$ vector model. This is conjectured to be dual to higher-spin Vassiliev gravity in four dimensions [31]. Higher-spin theories are very different from Einstein gravity. However, ample evidence indicates that the behavior of the free energy of vector models qualitatively captures that of duals to Einstein gravity when one restricts to scalar, vector or spin 2 deformations [32–34]. This includes a remarkable qualitative agreement of the relation between the vev and the source for the particular scalar potential (2.1) [36]. We therefore view these vector models as dual toy models of eternal inflation and proceed to evaluate their partition functions.

Specifically we consider the $O(N)$ vector model on squashed deformations of the three-sphere,

$$ds^2 = \frac{r_0^2}{4}\left((\sigma_1)^2 + \frac{1}{1+A}(\sigma_2)^2 + \frac{1}{1+B}(\sigma_3)^2\right) , \tag{2.4}$$

where r_0 is an overall scale and σ_i, with $i = 1, 2, 3$, are the left-invariant one-forms of $SU(2)$. Note that the Ricci scalar $R(A, B) < 0$ for large squashings [33]. We further turn on a mass deformation $\mathcal{O}$ with coupling α. This is a relevant deformation which in our dual $O(N)$ vector toy model induces a flow from the free to the critical $O(N)$ model. The coefficient α is imaginary in the dS domain of the wave function as discussed above. Hence we are led to evaluate the partition function, or free energy, of the critical $O(N)$ model as a function of the squashing parameters A and B and an imaginary mass deformation $\alpha \equiv \tilde{m}^2$. The key question of interest is whether or not the resulting holographic measure (1.1) favors large deformations as semiclassical gravity would lead one to believe.

The deformed critical $O(N)$ model is obtained from a double trace deformation $f(\phi \cdot \phi)^2/(2N)$ of the free model with an additional source $\rho f \tilde{m}^2$ turned on for the single trace operator $\mathcal{O} \equiv (\phi \cdot \phi)$. By taking $f \to \infty$ the theory flows from its unstable UV fixed point, where the source has dimension one, to its critical fixed point with a source of dimension two [31]. To see this we write the mass deformed free model partition function as

$$Z_{\text{free}}[m^2] = \int \mathcal{D}\phi e^{-I_{\text{free}} + \int d^3x\sqrt{g}m^2\mathcal{O}(x)} , \tag{2.5}$$

where I_{free} is the action of the free $O(N)$ model

$$I_{\text{free}} = \frac{1}{2}\int d^3x\sqrt{g}\left(\partial_\mu\phi_a\partial^\mu\phi^a + \frac{1}{8}R\phi_a\phi^a\right) . \tag{2.6}$$

Here ϕ_a is an N-component field transforming as a vector under $O(N)$ rotations and R is the Ricci scalar of the squashed boundary geometry. Introducing an auxiliary variable

$\tilde{m}^2 = \frac{m^2}{\rho f} + \frac{\mathcal{O}}{\rho}$ yields

$$Z_{\text{free}}[m^2] = \int \mathcal{D}\phi \mathcal{D}\tilde{m}^2 e^{-I_{\text{free}} + N\int d^3x\sqrt{g}\left[\rho f \tilde{m}^2 \mathcal{O} - \frac{f}{2}\mathcal{O}^2 - \frac{1}{2f}(m^2 - \rho f \tilde{m}^2)^2\right]} , \tag{2.7}$$

which can be written as

$$Z_{\text{free}}[m^2] = \int \mathcal{D}\tilde{m}^2 e^{-\frac{N}{2f}\int d^3x\sqrt{g}(m^2 - \rho f \tilde{m}^2)^2} Z_{\text{crit}}[\tilde{m}^2] , \tag{2.8}$$

with

$$Z_{\text{crit}}[\tilde{m}^2] = \int \mathcal{D}\phi e^{-I_{\text{free}} + N\int d^3x\sqrt{g}\left[\rho f \tilde{m}^2 \mathcal{O} - \frac{f}{2}\mathcal{O}^2\right]} . \tag{2.9}$$

Inverting (2.8) gives Z_{crit} as a function of Z_{free}:

$$Z_{\text{crit}}[\tilde{m}^2] = e^{\frac{Nf\rho^2}{2}\int d^3x\sqrt{g}\tilde{m}^4} \int \mathcal{D}m^2 e^{N\int d^3x\sqrt{g}\left(\frac{m^4}{2f} - \rho\tilde{m}^2 m^2\right)} Z_{\text{free}}[m^2] . \tag{2.10}$$

The value of ρ can be determined by comparing two point functions in the bulk with those in the boundary theory [34]. For the $O(N)$ model this implies $\rho = 1$, which agrees with the transformation from critical to free in [35].

We compute Z_{crit} for a single squashing $A \neq 0$ and $\tilde{m}^2 \neq 0$ by first calculating the partition function of the free mass deformed $O(N)$ vector model on a squashed sphere and then evaluate (2.10) in a large N saddle point approximation[2]. Evaluating the Gaussian integral in (2.5) amounts to computing the following determinant

$$-\log Z_{\text{free}} = F = \frac{N}{2}\log\left(\det\left[\frac{-\nabla^2 + m^2 + \frac{R}{8}}{\Lambda^2}\right]\right) , \tag{2.11}$$

where Λ is a cutoff that we use to regularize the UV divergences in this theory. The eigenvalues of the operator in (2.11) can be found in closed analytic form [37],

$$\lambda_{n,q} = n^2 + A(n-1-2q)^2 - \frac{1}{4(1+A)} + m^2 , \quad q = 0, 1, \ldots, n-1, \; n = 1, 2, \ldots \tag{2.12}$$

To regularize the infinite sum in (2.11) we follow [33, 34] and use a heat-kernel type regularization. Using a heat-kernel the sum over eigenvalues divides in a UV and an IR part. The latter converges and can readily be done numerically. By contrast the former contains all the divergences and should be treated with care. We regularize this numerically

[2] The generalization to double squashings $A, B \neq 0$ yields qualitatively similar results but requires extensive numerical work and is discussed in [36].

by verifying how the sum over high energy modes changes when we vary the energy cutoff. From a numerical fit we then deduce its non-divergent part which we add to the sum over the low energy modes to give the total renormalized free energy. The resulting determinant after heat-kernel regularization captures all modes with energies lower than the cutoff Λ. The contribution of modes with eigenvalues above the cutoff is exponentially small. For more details on this procedure we refer to [33, 36].

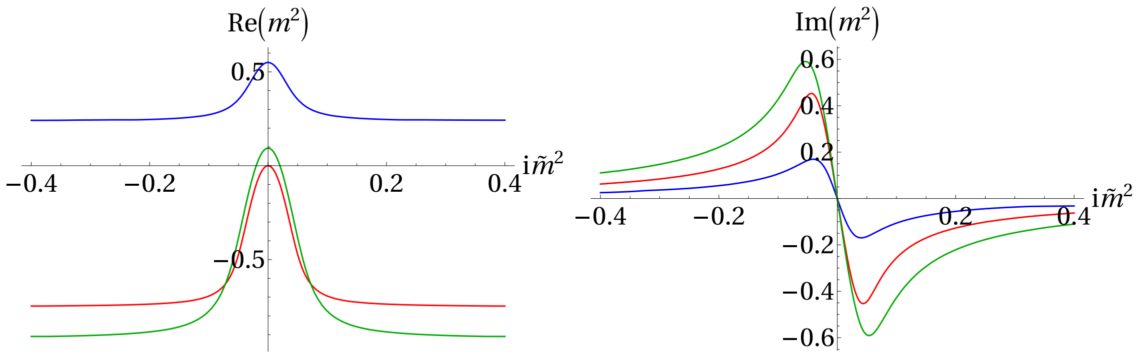

FIG. 2: The real and imaginary parts of the solutions m^2 of the saddle point equation (2.13) are shown for three different values of a single squashing, i.e. $A = -0.8$ (blue), $A = 0$ (red) and $A = 2.06$ (green). For large $i\tilde{m}^2$ we have $\mathrm{Re}(m^2) \to -R/8$.

To evaluate the holographic measure we must substitute our result for $Z_{\mathrm{free}}[A, m^2]$ in (2.10) and compute the integral in a large N saddle point approximation. The factor outside the path integral in (2.10) diverges in the large f limit. We cancel this by adding the appropriate counterterms. The saddle point equation then becomes

$$\frac{2\pi^2}{\sqrt{(1+A)(1+B)}}\left(\frac{m^2}{f} - \tilde{m}^2\right) = -\frac{\partial \log Z_{\mathrm{free}}[m^2]}{\partial m^2} \ . \tag{2.13}$$

We are interested in imaginary $\tilde{m}^2$ as discussed above. This means we need $Z_{\mathrm{free}}[A, m^2]$ for complex deformations m^2. Numerically inverting (2.13) in the large f limit we find a saddle point relation $m^2(\tilde{m}^2)$. This is shown in Fig. 2, where the real and imaginary parts of m^2 are plotted as a function of $i\tilde{m}^2$ for three different values of A.

Notice that $\mathrm{Re}(m^2) \geq -R(A)/8$. This reflects the fact that the determinant (2.11), which is a product over all eigenvalues of the operator $-\nabla^2 + m^2 + R/8$, vanishes when the operator has a zero eigenvalue. Since the lowest eigenvalue of the Laplacian ∇^2 is always zero, the first eigenvalue λ_1 of the operator in (2.11) is zero when $R/8 + m^2 = 0$. In the region of configuration space where the operator has one or more negative eigenvalues the Gaussian

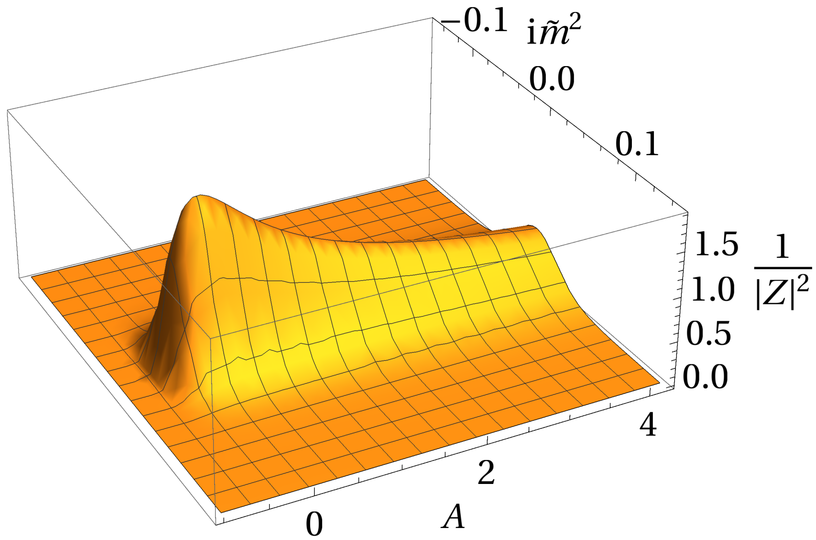

FIG. 3: The holographic probability distribution in a dual toy model of eternal inflation as a function of the coupling of the mass deformation $\tilde{m}^2$ that is dual to the bulk scalar, and the squashing A of the future boundary that parameterizes the amount of asymptotic anisotropy. The distribution is smooth and normalizable over the entire configuration space and suppresses strongly anisotropic future boundaries.

integral (2.5) does not converge, and (2.11) does not apply. This in turn means that the holographic measure $Z^{-1}_{crit}[A,\tilde{m}^2]$ is zero on such boundary configurations, as we now see.

Inserting the relation $m^2(\tilde{m}^2)$ in (2.10) yields the partition function $Z_{\rm crit}[A,\tilde{m}^2]$. We show the resulting two-dimensional holographic measure in Fig. 3. The distribution is well behaved and normalizable with a global maximum at zero squashing and zero deformation corresponding to the pure de Sitter history, in agreement with the F–theorem and its spin-2 extensions. When the scalar is turned on the local maximum shifts slightly towards positive values of A. However the total probability of highly deformed boundary geometries is exponentially small as anticipated[3]. We illustrate this in Fig. 4 where we plot two one-dimensional slices of the distribution for two different values of $\tilde{m}^2$.

[3] The distribution has an exponentially small tail in the region of configuration space where the Ricci scalar $R(A)$ is negative and $Z_{\rm free}$ diverges. We attribute this to our saddle point approximation of (2.10).

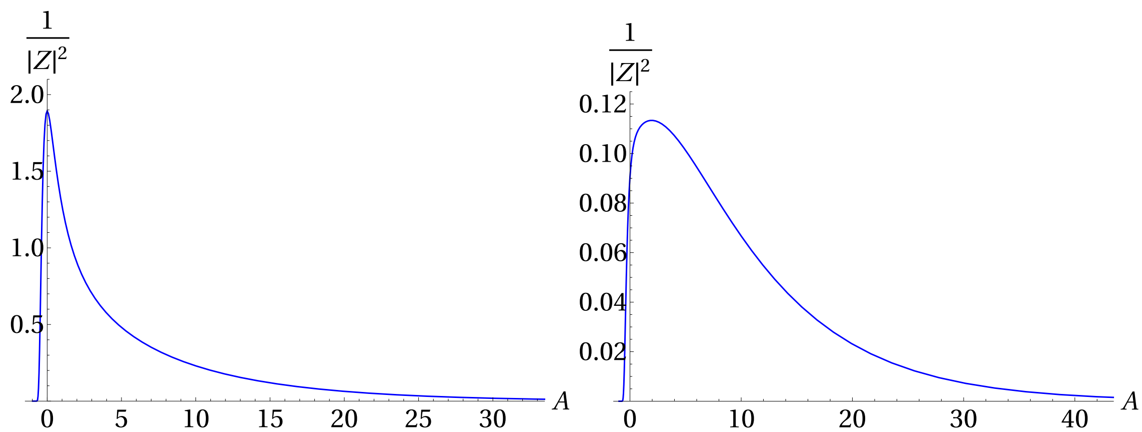

FIG. 4: Two slices of the probability distribution for $\tilde{m}^2 = 0.0$ (left) and $\tilde{m}^2 = 0.05i$ (right).

C. Global measure: general metric deformations

It is beyond the current state-of-the-art to evaluate partition functions in vector models for more general large metric deformations. However the above calculation suggests that the amplitude of such deformations is highly suppressed in the holographic measure.

This is because the action of any dual CFT includes a conformal coupling term of the form $R\phi^2$. For geometries that are close to the round sphere this is positive and prevents the partition function from diverging [39]. On the other hand the same argument suggests that the conformal coupling likely causes the partition function to diverge on boundary geometries that are far from the round conformal structure. These include in particular geometries with patches of negative curvature or, more accurately, a negative Yamabe invariant[4].

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[4] The Yamabe invariant $Y(\tilde{h})$ is a property of conformal classes. It is essentially the infimum of the total scalar curvature in the conformal class of $\tilde{h}$, normalized with respect to the overall volume. It is defined as

$$Y(\tilde{h}) \equiv \inf_\omega \mathcal{I}(\omega^{1/4}\tilde{h}) \tag{2.14}$$

where the infimum is taken over conformal transformations $\omega(x)$ and $\mathcal{I}(\omega\tilde{h})$ is the normalized average scalar curvature of $\omega^{1/4}\tilde{h}$,

$$\mathcal{I}(\omega^{1/4}\tilde{h}) = \frac{\int_M \left(\omega^2 R(\tilde{h}) + 8(\partial\omega)^2\right)\sqrt{\tilde{h}}\, d^3x}{\left(\int_M \omega^6 \sqrt{\tilde{h}}\, d^3x\right)^{1/3}} \tag{2.15}$$

There always exists a conformal transformation $\omega(x)$ such that the metric $\tilde{h}' = \omega^{1/4}\tilde{h}$ has constant scalar curvature [40]. The infimum defining Y is obtained for this metric $\tilde{h}'$.

The Yamabe invariant is negative in conformal classes with a metric of constant $R < 0$. Since the lowest eigenvalue of the conformal Laplacian is negative on such backgrounds this means that the partition function of a CFT does not converge, thereby strongly suppressing the amplitude of such conformal classes in the measure (1.1). Our result for the holographic measure specified by the partition function of the deformed $O(N)$ model on squashed spheres provides a specific example of this, as discussed above.

Conformal classes with negative $Y(\tilde{h})$ include the highly irregular constant density surfaces from eternal inflation featuring in a semiclassical gravity analysis. This general argument therefore suggests their amplitude will be low in a holographic measure. We interpret this as evidence against the idea that eternal inflation typically leads to a highly irregular universe with a mosaic structure of bubble like patches separated by inflationary domains[5].

III. DISCUSSION

We have used gauge-gravity duality to describe the quantum dynamics of eternal inflation in the no-boundary state in terms of a dual field theory defined on a global constant density surface in the large volume limit. Working with the semiclassical form (1.1) of dS/CFT the field theories are Euclidean AdS/CFT duals deformed by a low dimension scalar operator that is sourced by the bulk scalar driving eternal inflation.

The inverse of the partition function specifies the amplitude of different shapes of the conformal boundary surface. This yields a holographic measure on the global structure of eternally inflating universes. We have computed this explicitly in a toy model consisting of a mass deformed interacting O(N) vector theory defined on squashed spheres. In this model we find that the amplitude is low for geometries far from the round conformal structure. These include surfaces with negative scalar curvature which, we have argued, are strongly suppressed in a holographic measure in general. Based on this we conjecture that eternal inflation produces universes that are relatively regular on the largest scales. This is radically different from the usual picture of eternal inflation arising from a semiclassical gravity

[5] This resonates with [9] where we argued that probabilities for local observations in eternal inflation can be obtained by coarse-graining over the large-scale fluctuations associated with eternal inflation, thereby effectively restoring smoothness. Our holographic analysis suggests that the dual description implements some of this coarse-graining automatically.

treatment.

Our conjecture strengthens the intuition that holographic cosmology implies a significant reduction of the multiverse to a much more limited set of possible universes. This has important implications for anthropic reasoning. In a significantly constrained multiverse discrete parameters are determined by the theory. Anthropic arguments apply only to a subset of continuously varying parameters, such as the amount of slow roll inflation.

However, the setup we have considered does not allow us to describe the transition from the quantum realm of eternal inflation to a universe in the semiclassical gravity domain. This is because our duals are defined in the UV and live at future infinity. It therefore remains an open question whether the conjectured smoothness of global constant density surfaces impacts the eternity of eternal inflation[6]. To answer this will require a significant extension of holographic cosmology to more realistic cosmologies. It has been suggested that in such models, inflation corresponds to an IR fixed point of the theory [22]. The detailed exit from inflation will then be encoded in the coupling between the field theory degrees of freedom and the bulk dynamics in a manner somewhat analogous to the holographic description of vacuum decay in AdS [41].

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