HYDROSTATIC EQUILIBRIUM and PLANETARY DIFFERENTIATION

Mike Luciuk, Jeremy Carlo

INTRODUCTION

During formation of the solar system 4.6 billion years ago, a cold molecular gas and dust cloud began to contract around its denser portions. As the cloud contracted, its rotation rate increased and it evolved to a disk-like configuration. Over time, the proto-Sun and proto-planets formed. Figure 1 illustrates planet compositions and temperatures versus proto-Sun distance. As illustrated, four terrestrial planets and many smaller rocky bodies were formed closest to the proto-Sun, then as temperatures fell with distance, the four gas giants. Finally, beyond Neptune, many small bodies formed composed of a substantial percentage of various ices. The temperature decrease with proto-Sun distance resulted in significant composition variations of these three classes of solar system bodies. The resulting variations in mass, composition, and density produced major differences in states of hydrostatic equilibrium and differentiation.

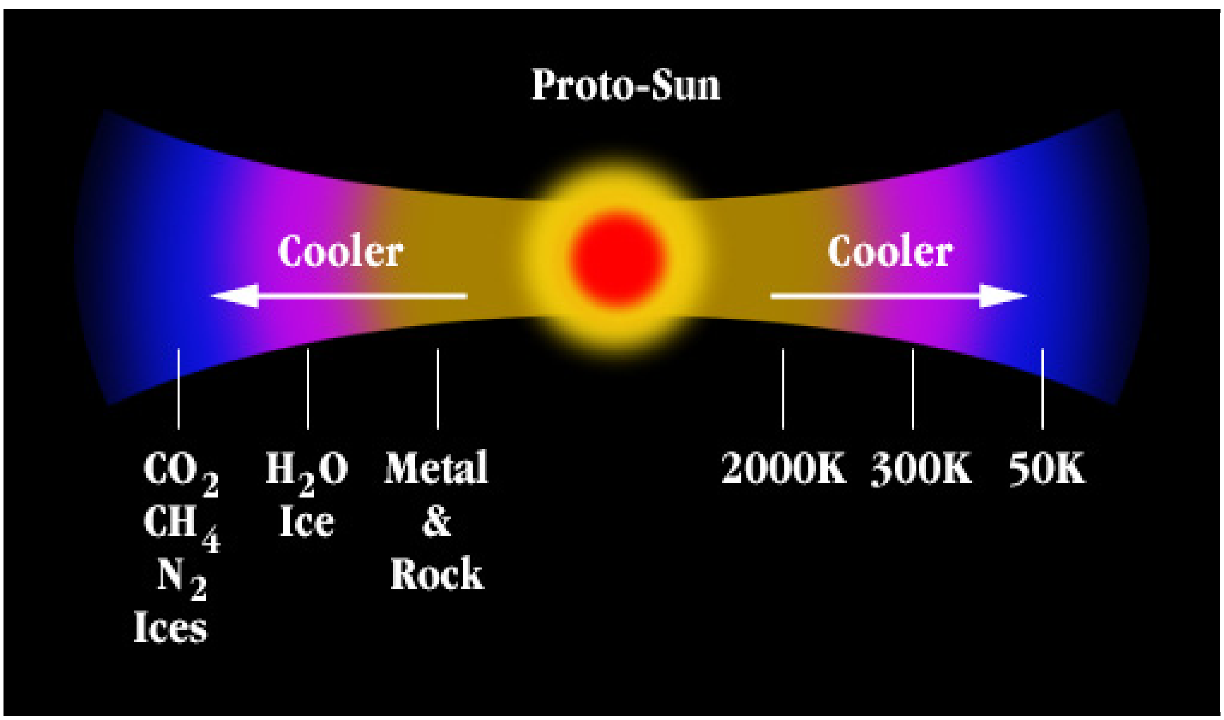

FIGURE 1. Early Solar System Planet Formation Environment
http://lasp.colorado.edu/~bagenal/1010/SESSIONS/11.Formation.html

HYDROSTATIC EQUILIBRIUM

An astronomical body is in a state of hydrostatic equilibrium (HE) when its self gravitational force is balanced by its internal pressure; the body is neither expanding nor contracting. From a technical perspective, a body in HE will assume a spherical shape to minimize gravitational potential since any deviations from sphericity increases gravitational energy. The size of deviations is affected by the body's mass; the larger the mass, the smaller the deviations must be, all else being equal. However, its actual form depends on its rotation rate, mass, material strength, geologic activity and its history. Figure 2 illustrates the balance between the two forces with an equation quantifying their magnitudes.

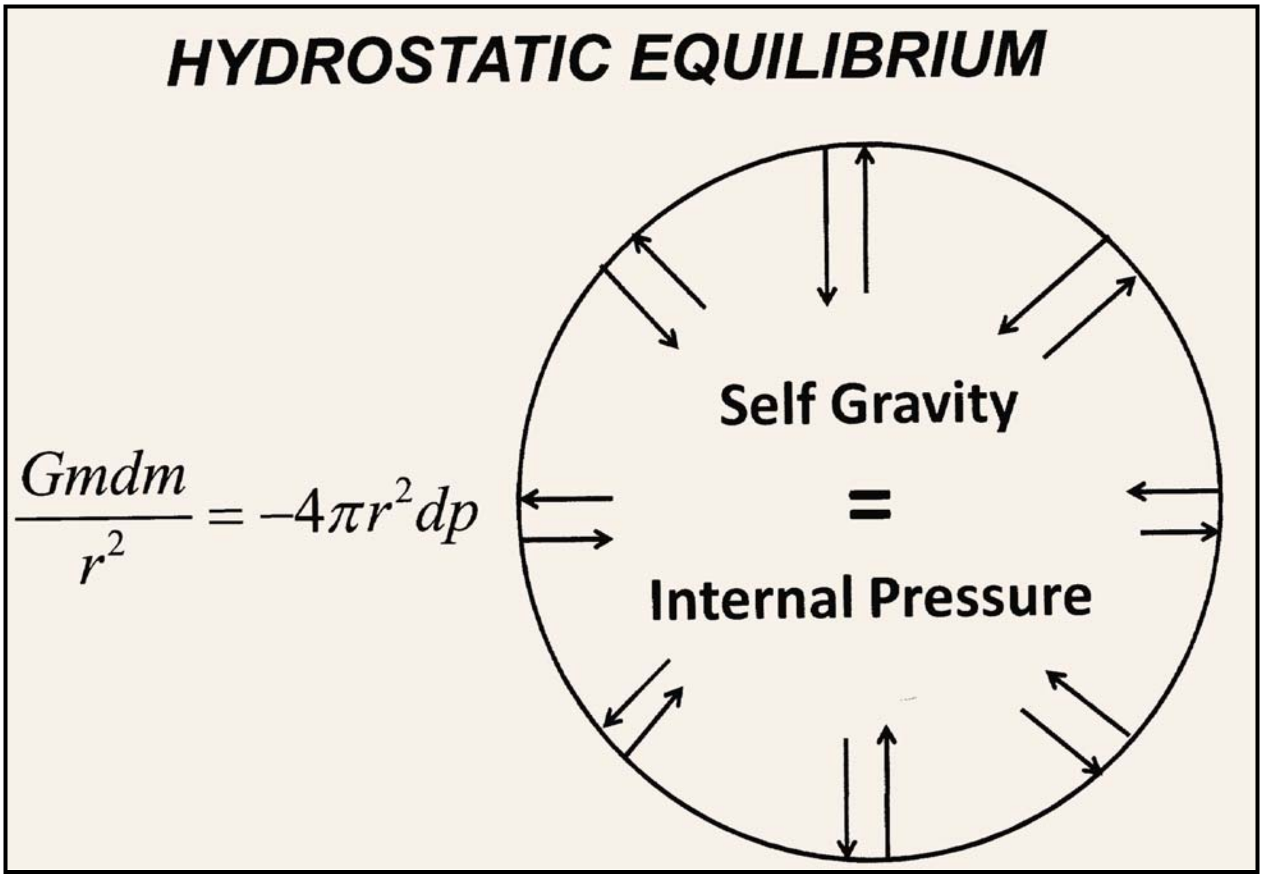

FIGURE 2. Balance of Forces in Hydrostatic Equilibrium

Solving the equation in Figure 2 for the pressure at the center of a body in HE,

$$P_c = \frac{3GM^2}{8\pi R^4} \qquad (1)$$

Where P_c is the pressure at the center in pascals, newtons/m^2.

(1 atmosphere = 101,325 pascals)

G is the gravitational constant, 6.673 x 10^{-11} $m^3/kg\ s^2$

M is the body's mass in kg.

R is the body's radius in m.

Equation 1 only gives approximate central pressure because it assumes a constant density throughout the body. It can offer rough results for terrestrial bodies, but badly underestimates pressures in gaseous planets or the Sun. The equation forecasts Earth's central pressure to be about 1.7 million atmospheres, which is about half the value estimated by more sophisticated methods. The equation predicts Pluto's central pressure to be about 6,440 atmospheres, about 1/250th forecasted for Earth.

Wikipedia has an interesting article listing all known solar system bodies in HE (http://en.wikipedia.org/wiki/List_of_Solar_System_objects_in_hydrostatic_equilibrium). They include the Sun, eight major planets, five dwarf planets, and nineteen planetary satellites. Also listed are ten plutoids, possibly large enough to be in HE, but not yet recognized by the IAU as dwarf planets. The largest solar system body listed is the Sun, of course. The smallest is Mimas, a satellite of Saturn whose spheroidal form has been shaped by the planet's tidal forces. Mimas is an icy body with a diameter of about 400 km and density slightly greater than water.

It can be difficult to determine the shape of distant KBOs by optical imaging. In 2008, the IAU announced that a TNO with an absolute magnitude of +1 or brighter has sufficient size to be in HE and be classified as a plutoid. Recall that a body's absolute magnitude H, is the visual magnitude an observer would record if the asteroid was located 1 astronomical unit (AU) from Earth and 1 AU from the Sun at a zero phase angle. Astronomers can estimate TNO sizes based on H magnitudes and assumptions of albedo. Table 1 shows diameter (km) by absolute magnitude and differing albedo assumptions.

H	Albedo 0.50	Albedo 0.25	Albedo 0.05
-2.0	4,700	6,700	14,900
0	1,900	2,600	5,900
+1.0	1,200	1,700	3,700
+2.0	750	1,050	2,400
+4.0	300	420	940
+6.0	120	170	370

+8.0	45	65	150

TABLE 1. Asteroid Sizes (km)
http://cfa-www.harvard.edu/iau/lists/Sizes.html

Let's examine some factors that determine whether an astronomical body can achieve a spherical shape, the hallmark of HE. One key factor is the bulk modulus, the strength of materials that make up an astronomical body. Bulk modulus measures the ability of a substance to resist compressive forces. The bulk modulus of rocky material is about an order of magnitude larger than ice, which means it's easier for an icy body to achieve HE than a rocky body. Mimas is in HE with a 400 km diameter, while Ceres, the smallest dwarf planet body of mostly rocky composition in HE has a mean diameter of about 940 km. Pallas (~610 km) and Vesta (~540 km) are moderately elongated.

The most important HE requirement is mass. Comets' low mass (~10^{15} kg) gravitational forces hold constituent ice and rock material together like a rubble pile. Mimas' mass of 3.7×10^{19} kg mass coupled with ice's lower bulk modulus permits its HE form. Ceres' 9.43×10^{20} kg mass is sufficient to overcome its higher rocky bulk modulus and achieve HE. Vesta and Pallas have masses 3.5x – 4.5x respectively smaller than Ceres, resulting in definite ellipsoid shapes. In a recent arxiv paper Lineweaver and Norman), the authors determined that icy bodies can achieve HE with about 400 km diameters, and rocky asteroids at about 600 km based in first principles.

To summarize, a non-rotating astronomical body with sufficient mass so its self-gravity can overcome its materials' strength will acquire a spheroidal shape and achieve hydrostatic equilibrium. This tends to occur as bodies made of ice approach sizes of about 400 km and those of about 600 km made of rock. Vesta and Pallas were in transition toward HE during their formation, but fell somewhat short of full sphericity.

DIFFERENTIATION

A good description of planetary differentiation similar to what has occurred on Earth (Figure 3) follows (http://wapi.isu.edu/Geo_Pgt/Mod03_PlanetaryEvo/mod3_pt1.htm):

> Planetary differentiation … refers to the processes that cause an essentially homogeneous accreted body that is made up of primordial solar material to become separated into layers having different chemical and/or physical properties. If a planetary body is large enough it will develop a core, mantle and crust each of which may be further subdivided.

Differentiation operates as materials of varying density are separated by a body's self gravity, with those of the highest density moving to its center. Melting or partial melting of material is required for the process to occur, which takes place over long time scales. The rate of differentiation depends on buoyancy forces and heat generated in the materials which may be solid, but exhibit fluid properties over geologic time. This can be quantified by the Rayleigh number, a complex fluid mechanics relationship, the value of which determines the onset of convection, the prime mover of differentiation.

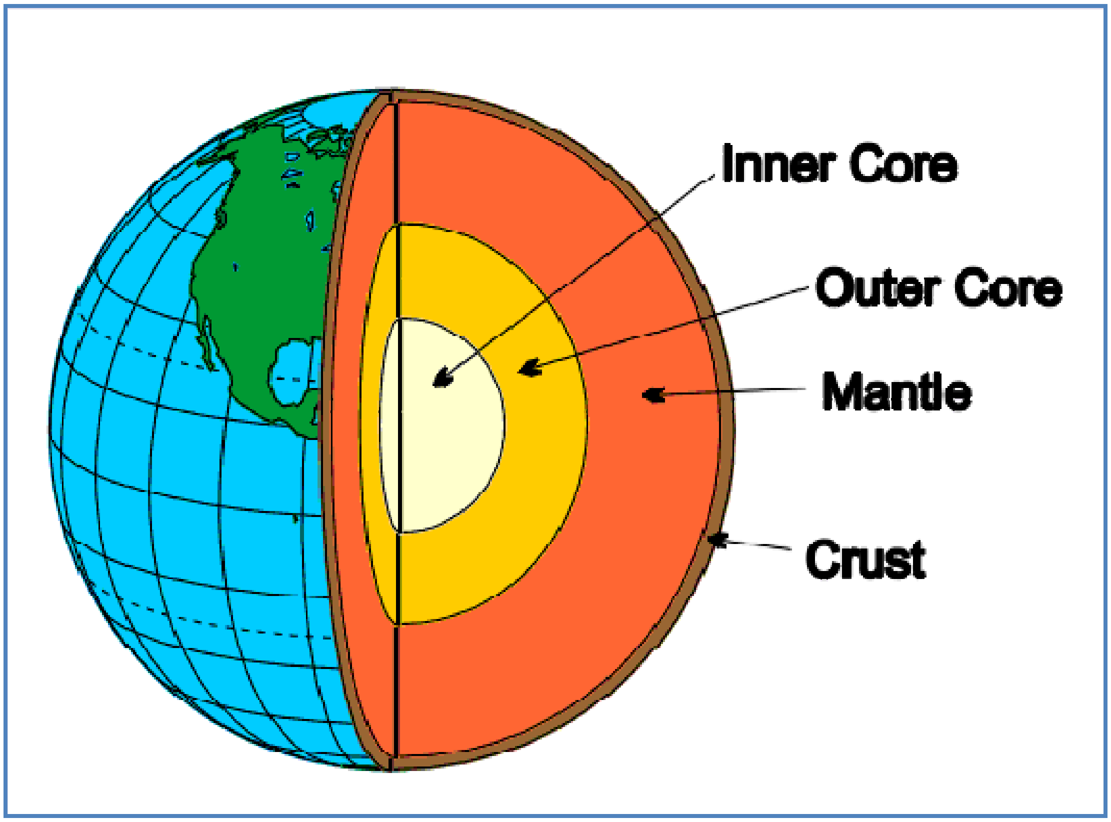

FIGURE 3. Earth's differentiated Layers
http://www.didcotgirls.oxon.sch.uk/depts/geog/bcs_geography/images/earth%20structure3.gif

There are several sources of heat relating to differentiation, but planetary scientists think that most important are those from radioactive elements. Relatively short half life (0.7–1.5 million years) isotopes like ^{26}Al and ^{60}Fe were responsible for initial internal heating creating differentiation early in planetary evolution, and especially important for smaller bodies like asteroids. This means that dust in the solar system molecular cloud contained material from nearby red giant, nova or supernova events occurring at planetoid formation time. Long lived isotopes like ^{238}U and ^{40}K with half lives in the billions of years were important differentiation sources for larger bodies like Earth, and continue to heat interiors to the present day. During differentiation, as denser material descended to the center, heat was generated by the exchange of gravitational potential energy to heat. An important differentiation heat source comes from tidal heating of some planetary satellites. The volcanism exhibited by Jupiter's Io comes from tidal effects on the satellite. Of course, heat also occurs from solar radiation and impacts from space detritus, but these sources tend to mainly affect surface temperatures, and are mostly unrelated to aiding differentiation.

There is ample meteorite evidence that asteroids have undergone differentiation. Spectral characteristics of differentiated meteorites such as achondrites have been matched to Vesta, which confirms its differentiated history as well as of other asteroids. Figure 4 shows an achondrite meteorite found in Antarctica. The heat sources were short half life isotopes based on analysis of the meteorites, which reveal the presence of the stable daughter products of ^{26}Al and ^{60}Fe isotopes. It's thought that during formation, many asteroids reached internal temperatures (~1,500 K) capable of melting minerals resulting in a differentiated structure. The *Dawn Mission* reaching Vesta in 2011, and Ceres in 2015 will reveal surface detail of these asteroids and evidence of past geologic activity.

FIGURE 4. Achondritic Meteor found in Antarctica
http://www.solarviews.com/cap/meteor/achndrit.htm

Less well known is the capability of KBOs to achieve that state given their different composition and great solar distances. KBOs are thought to be made up of about 50% of various ices and are located in ~50 K temperatures. There is no reason to believe that KBOs in HE couldn't become differentiated, since in their formation it's likely they were also heated by short and long half life isotopes. The prominence of crystalline ice on some KBOs, which forms at 110 K versus amorphous ice, is a possible indication of geologic activity associated with differentiation. We'll have to wait for images from *New Horizons* in 2015 for surface confirmation of KBO differentiation.

CONCLUDING REMARKS

Are hydrostatic equilibrium and differentiation related? Can we assume that a body in HE has achieved differentiation? When a body made of ice (Mimas) achieves HE, there is nothing to differentiate since it's made up of a single compound, but may have different phases internally. Mark Sykes, the well known planetary scientist claims in support for Pluto's planetary status that

> Objects become "round" when they are so massive that their gravity crushes them into a shape that is in hydrostatic equilibrium. Heat from formation and the decay of radionuclides increases interior temperatures to the point where differentiation, mantle convection, and other processes occur. See: http://www.sciencemag.org/cgi/content/full/319/5871/1765?ijkey=exNqd2F83NhIM&keytype=ref&siteid=sci

Extreme pressures at the center of a body in HE may result in sufficient heat generation to create melting and adding to heat from radioactive isotopes promoting differentiation.

On the other hand, we cannot assume that all differentiated bodies achieve HE. Even if small asteroids are differentiated, it's unlikely they have sufficient mass to reach spherical form. So differentiation does not necessarily imply a body has achieved HE.

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