Structural integrity of Notre Dame Cathedral after the fire of April 15th, 2019

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Abstract

We consider the consequences of the fire destructions, happened on April 15th, 2019, on the static regime of the Cathedral Notre Dame of Paris. In particular, we ponder the effects of the wind on the stability of the Cathedral at its post-fire state and compare them with those calculated, with the same numerical model and assumptions before fire. We show that a major consequence of the fire destruction is the considerable reduction of the wind strength of the Cathedral.

Keywords: Notre Dame; Wind strength; Gothic structures.

1. Introduction

The fire of April 15th, 2019, has destroyed a large part of the structure of the Cathedral Notre Dame of Paris. The whole original roofing structure, composed by the wood of, approximately, 1300 oaks and covered with 210 tons of lead tiles, has been entirely destroyed by the fire, as well as the spire. The intensity of the fire, that has produced high temperatures, has also damaged the stone structure: a large part of the sexpartite rib vault has collapsed, in the main aisle, in the transept and also at the center of the Cathedral, where choir, aisle and transept meet together. It is also possible that future analyses may reveal important damage of the stone parts that were more exposed to fire, having as a consequence the decrease of the strength of the stone. All these facts have certainly engendered changes in the static regime of the Cathedral. This is important for different reasons: on one hand, the restoration of the Cathedral will last, probably, several years, so the new static regime has a rather permanent character. On the other hand, it is important, for all the duration of the restoration phase, to assess the vulnerability of the Cathedral to different actions, in order avoid further collapse and adequately reinforce the structure.

A gothic Cathedral is a rather complicate structure, and in particular Notre Dame of Paris, a church with five aisles, two rows of lateral chapels and large tribunes, see Fig 1. The equilibrium of such a structure is determined by the mutual interactions of all of its parts: pillars, buttresses, vaults, flying buttresses, choir, towers and also the timber structure of the roofing, all together collaborating to ensure the statical equilibrium of the whole construction and to withstand the two main actions to which a Cathedral is submitted: the vertical action of its own weight and the horizontal one due to wind. In particular, the weight of the roofing, estimated at about 700 tons (including the weight of the lead tiles) served as a ballast contributing to withstand the horizontal wind forces; it also connected together the two guttering walls

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at the top of the main aisle, helping to transmit the action of the wind from the windward side to the leeward one. The long-span ($\sim$ 12 m) flying buttresses provided a thrust at the level of the high vault to counterbalance its outward thrust.

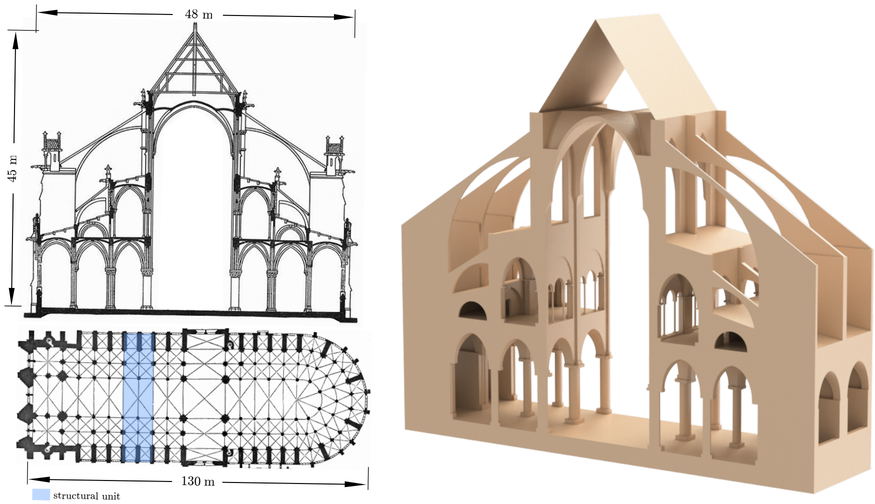

Figure 1: The Cathedral of Notre-Dame in Paris, as it was before the fire. From left to right: section (top) and plan view (bottom), numerical model (overall view). We model the blue-shadowed area, which corresponds to a part of the high vault that has collapsed after the fire.

In the present situation, all the timber structure of the roofing has burnt and large parts of the high vault have collapsed. Hence, the transmission of the forces, at the level of the clearstorey, is deeply altered. The thrust exerted by the flying buttresses is no more counterbalanced by the high vault and the equilibrating ballast of the roof has disappeared. Moreover, when the wind acts on the wall of the clearstorey, its action adds to that of the flying buttresses, producing an overturning moment at its base that can lead to its collapse. In addition, due to the lack of the wooden structure at the top of the guttering walls, the action of the wind is no more transmitted from the windward wall to the leeward one, that, together with the flying buttresses on its side, were equilibrating the wind thrust. Finally, the wall of the clearstorey directly hit by the blow can collapse for a sufficiently strong wind. The question is hence: which is the highest wind speed that the Cathedral can withstand in its present state before a major collapse of its upper structure? We try to give an answer to this question, using the same nonlinear approach that we used in a recent paper, [1]. To have a correct comparison between the two states of the Cathedral, i.e. before and after the fire, we use exactly the same numerical model and mechanical assumptions used in the cited paper. All these points are detailed in the sections below.

2. The numerical model

The analysis has been performed using the same finite element model with [1], by removing the high vault and the roof, see Fig. 2. For what concerns the material, we still model the stone structure through a nonlinear constitutive law, and in particular:

- in compression, the material is modeled as isotropic linearly elastic with infinite strength, an assumption often used in such situations, [2], [3];
- in tension, the material is assumed to be isotropic linearly elastic until the maximum principal stress does not exceed the tensile strength; a tensile strength $f_t = 0.5$ MPa is assumed for the material;
- when the maximum principal stress exceeds the tensile strength f_t, failure is modeled using a nonlinear constitutive law based on the softening model proposed in [4];

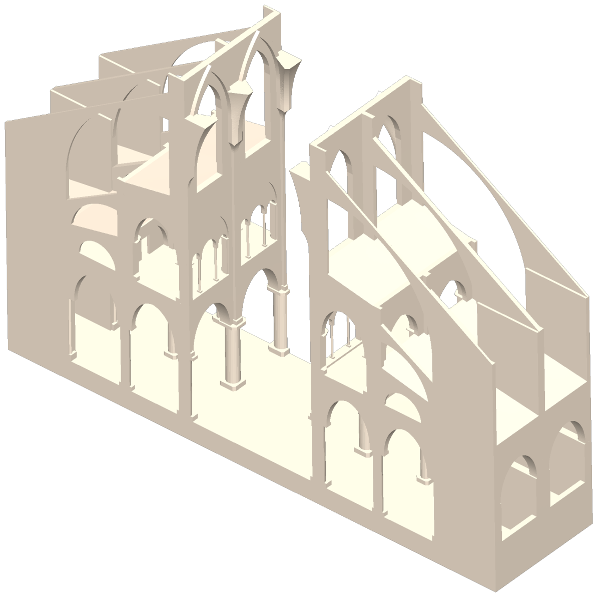

Figure 2: The numerical model of the Cathedral in the present state.

- an equivalent homogenized Young's modulus $E_{eq} = 14000$ MPa is assumed, [5];
- the Poisson's ratio is taken equal to $\nu_s = 0.25$;
- the density is evaluated at $\rho_s = 2000$ kg/m^3;
- the softening behavior is approximated by a bilinear law, characterized by a fracture energy $G_f = 11.3$ N/m.

Concerning the wind profile, like in other works on the wind strength of Gothic Cathedrals, [6], we have used the power law

$$v = v_0 \left(\frac{z}{z_0}\right)^{\alpha}, \tag{1}$$

with z_0 a reference height, where the wind speed v_0 is known, and α an exponent, set equal to 0.35, value suggested for urban areas, see [6]. Considering the skyline of Paris, we have chosen for z_0 the value of 10 m. The wind pressure p is then obtained as a drag force per unit of exposed surface using the relation

$$p = \frac{1}{2} C_D \ \rho \ v^2, \tag{2}$$

where ρ is the mass density of air, $\rho = 1.225$ kg/m^3 at an ambient temperature of 15°C, and C_D is the drag coefficient, that we have assumed to be equal to 1.5. Considering the situation of the Cathedral, after the fire destruction, and being interested in knowing the action of the wind on the clearstorey wall directly hit by the wind, it is sufficient to apply the pressure p on the windward side of the Cathedral, see Fig. 3.

As in [1], we identify the ultimate wind speed as the one at which the displacement of the top of the guttering wall becomes unbounded.

3. Effects of gravity and internal stress distribution change

We first consider the effects of the own weight of the structure and we compare the internal stress distribution between its current, damaged state and its intact one. This is done through an implicit, static Finite Element analysis.

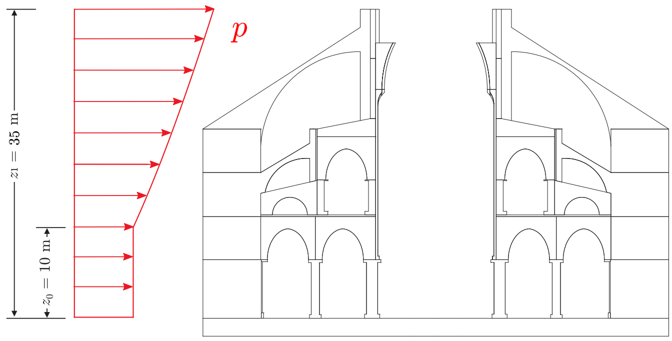

Figure 3: Wind pressure profile.

As it could be expected, the absence of the high vault produces an unbalanced thrust of the flying buttresses, that makes the clearstorey wall lean inward. The horizontal displacement of the top of the guttering wall, for the simple static action of gravity, is of 2.7 mm. At the same time, the maximum tensile stress in the flying buttresses attains the value of 0.25 MPa.

The situation is sketched in Fig. 4, where the maximum principal stresses are shown. It is apparent that, compared to the state of the Cathedral before the fire, the static regime has changed, especially in the flying buttresses, where now important tensile stresses appear.

These results reveal the strong impact that the destructions engendered by the fire have on the static regime of the Cathedral. More important are the effects related to the action of the wind, considered in the next Section.

4. Residual wind strength of the Cathedral

In [1], we evaluated the ultimate wind speed of the Cathedral at $v_0 = 222$ km/h. We consider now which is the residual wind strength of the Cathedral after the destructions made by the fire. A certain degree of approximation is intrinsic and ineradicable in this kind of analysis. However, given that we use the same numerical model and assumptions as in our previous study, the evaluation of the ultimate wind speed at the current, damaged state of the structure and its comparison with the ultimate wind strength at its previous, intact one, is a measure of the extent of damage and loss of structural integrity of the Cathedral.

The configuration resulting from the previous implicit analysis, considering only gravity, is now used as the starting point for a nonlinear explicit analysis in which the wind loads are applied in a quasi-static manner. A fictitious mass proportional damping is assumed in order to reach equilibrium rapidly and to dissipate unwanted oscillations (quasi-static condition). We evaluate the displacement of the top of the guttering wall for different wind speeds; when such a displacement becomes unbounded, this means that the Cathedral has reached its ultimate state: cracks have formed a collapse mechanism.

The result of the numerical simulations under wind and gravity actions are represented in Fig. 5. Here, we show the two curves relating the wind speed and the maximum horizontal displacement of the vault keystone, for the original state, and of the top of the guttering wall, for the damaged structure. From this Figure, it appears clearly that the highest wind speed that the Cathedral can now withstand is 90 km/h, i.e. 40% of the ultimate wind speed found when intact. In Fig. 6 we show the collapse mechanism of the Cathedral under the action of the wind and gravity.

In other words, the destructions due to the fire have reduced the wind strength of the Cathedral of about 60%. More important, is the fact that such a wind speed can be rather frequent in the Paris region, especially given the current climate deregulation. Consequently, it appears urgent the need for providing the Cathedral with temporary support structures, until the end of the restoration phase.

Notice that the real ultimate wind speed leading to collapse might be even lower than 90km/h for two reasons. First, the fire might have further reduced the mechanical strength of the stone and mortar, which leads to lower structural strength of the masonry. Second, our analysis does not take into account the

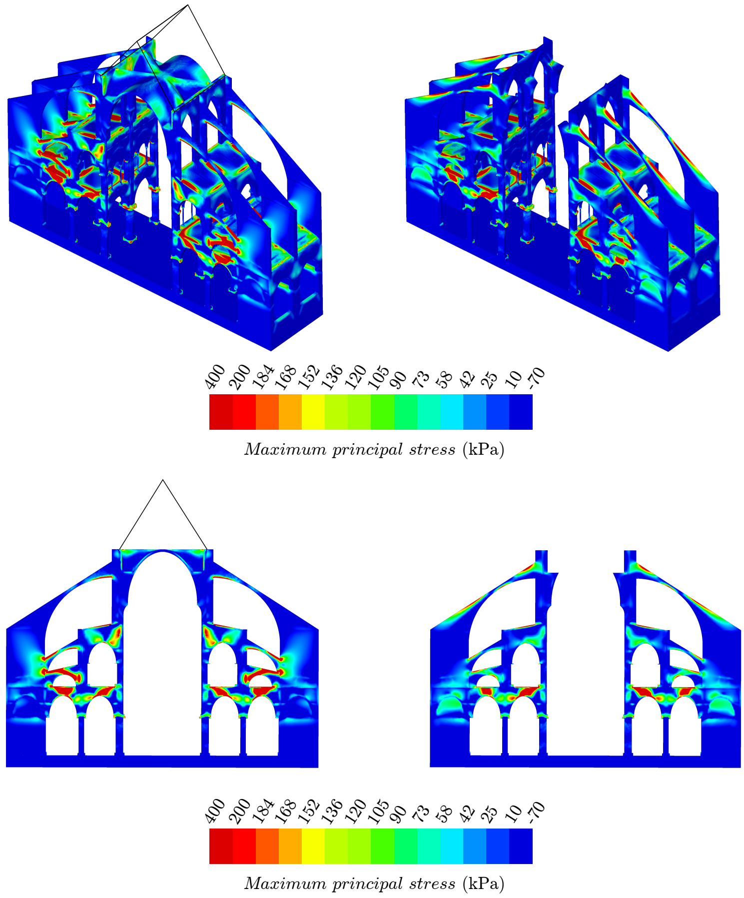

Figure 4: Maximum principal stress distribution, before (left) and after (right) the fire destructions.

windows, which can transfer additional loads to the structure. It is worth noticing that in our previous calculations, [1], we have supposed that for a wind as strong as 222 km/h, the windows are blown before the collapse of the Cathedral and therefore they do not transfer any mechanical loads to the masonry. However, this is not necessarily the case for a milder wind of 90 km/h.

5. Conclusion

Our analysis shows that the fire occurred on April 15th, 2019, in the roofing structure of Notre Dame of Paris has lead to an important stress redistribution under gravity loads. Moreover, the fire caused a reduction of about 60% of the strength of the Cathedral to wind actions. According to our calculations, the ultimate wind speed leading to collapse is now 90 km/h, instead of 222km/h that was before the fire. The real resistance of the structure to wind loads might be even lower, due to material degradation and the presence of the windows that can transfer additional wind loads to the masonry.

It is worth mentioning that this is a first study based on our previous analysis of the intact structure after removing the roof and the collapsed parts of the monument due to the fire. More detailed studies are of course necessary. Nevertheless, we believe that this work can provide a rational basis for immediate decision making regarding reinforcement and structural support. Moreover, it can help the authorities in the design of future interventions. The restoration and reconstruction of the monument is not a trivial task and it should take into account several aspects, including the principles set by the Venice Charter, in order to sustain the cultural and architectural value of Notre Dame of Paris.

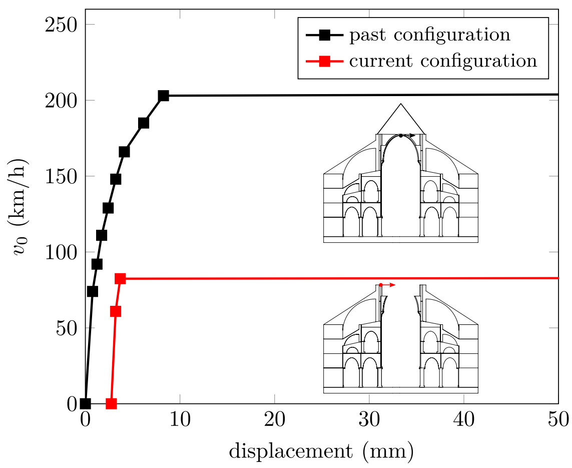

Figure 5: Wind speed v_0 versus horizontal displacement.

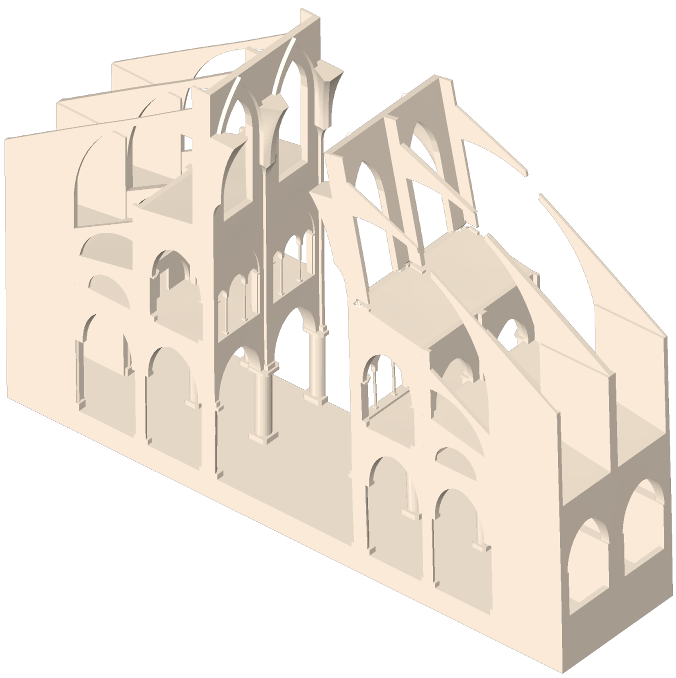

Figure 6: Collapse mechanism.

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