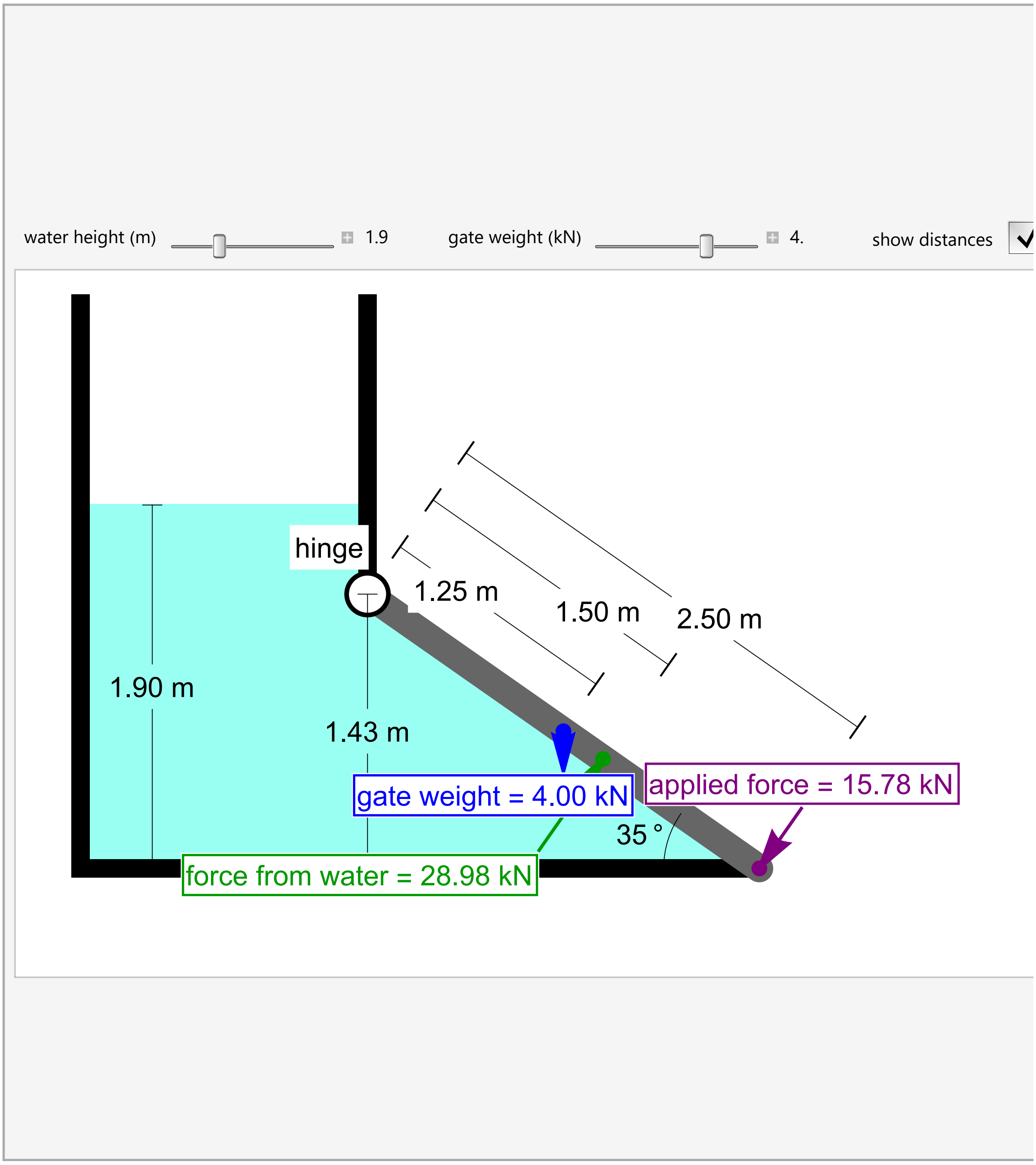
Forces on a Completely Submerged Gate



A gate hinged at the top is submerged under water. The gate is 2.5 meters long and 1 meter wide. This Demonstration calculates the force that must be applied to keep the gate closed. Use the sliders to set the water height and the gate weight. Check the "show distances" box to display distances.

THINGS TO TRY

Resize Images · Slider Zoom · Gamepad Controls · Automatic Animation

DETAILS

This Demonstration determines the magnitude of the applied force needed to keep a gate, which is w_2 meters wide and h_1 meters tall, closed (see Figure 1).

The magnitude of the resultant force is found by summing the differential forces over the gate surface:

$$F_R = \int_A \gamma\, h\, dA = \int_A \gamma\, y \sin\theta\, dA,$$

where $h = y\sin\theta$ is the vertical distance from the water surface (m), y is the y coordinate (along the diagonal) from the water surface (m), F_R is the resultant force (N), $A = L\,b$ is the gate area (m^2), b is the width of the gate (m), L is the length of the gate (m) as shown in Figure 1, θ is the angle (degrees) and γ is the specific weight of water (N/m^3).

Solving the integral for F_R:

$$F_R = \gamma\, h_c\, A = \gamma\, y_c \sin\theta\, A,$$

where $h_c = y_c \sin\theta$ is the vertical distance from the fluid surface to the centroid of the gate (m) and $y_c = D + L/2$ is the y coordinate of the gate centroid (m).

The y coordinate y_R of the resultant force can be found by summing moments around the hinge:

$$F_R\,(y_R - D) = \int_A (y - D)\, dF = \int_A \gamma\, y\,(y - D)\sin\theta\, dA.$$

When F_R is substituted into this equation, and the right side is integrated from D to $D + L$, then y_R becomes:

$$y_R = D + \frac{(D+L)^2\,(2\,L-D) + D^3}{6\,L y_c}.$$

A moment balance is done to determine the applied force that keeps the gate closed:

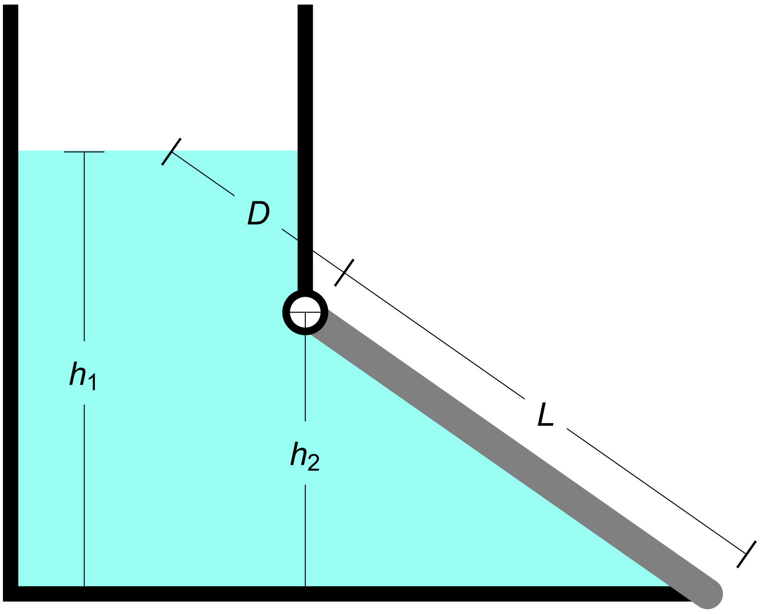
$$\sum M = 0 = -F_R\,(y_R - D) + W\,(y_c - D)\cos\theta + F_A\, L,$$

rearranging to solve for F_A:

$$F_A = \frac{F_R\,(y_R - D) - W\,(y_c - D)\cos\theta}{L},$$

where W is the weight of the gate (N), D is the diagonal length from the hinge to the top of the water (m) and F_A is the applied force (N).

Figure 1



Reference

[1] B. R. Munson, D. F. Young, T. H. Okiishi and W. W. Huebsch, *Fundamentals of Fluid Mechanics*, 6th ed., Hoboken, NJ: John Wiley and Sons, 2009 pp. 58–60.

RELATED LINKS

Forces on a Partially Submerged Gate (Wolfram Demonstrations Project)

PERMANENT CITATION

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"Forces on a Completely Submerged Gate"

http://demonstrations.wolfram.com/ForcesOnACompletelySubmergedGate/

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