

$$g_{\mu\nu} = \begin{bmatrix} \left(1-\frac{2GM}{rc^2}\right) & 0 & 0 & 0 \\ 0 & -\left(1-\frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{bmatrix}$$

Solutions of objects with angular momentum proved much more challenging to find as

$$g_{xx} = -1 \cdot \left(1 - 2 \cdot \frac{m}{r}\right)^{-1}$$

$$g_{tt} = 1 - \frac{2 \cdot m}{r}$$

$$c(r) = \sqrt{\frac{-g_{tt}}{g_{xx}}}$$

$$\textcolor{green}{c}(r) := \frac{\sqrt{1 - \frac{2 \cdot m}{r}}}{\sqrt{\left(1 - 2 \cdot \frac{m}{r}\right)^{-1}}}$$

$$x^2 + y^2 = r^2 \qquad \qquad \qquad m_{\text{zon}} := 1.98 \cdot 10^{30} \qquad \textcolor{green}{G} := 6.67 \cdot 10^{-11} \qquad \textcolor{green}{c} := 3 \cdot 10^8 \qquad r_{\text{zon}} := 680 \cdot 10^6$$

$$r = \sqrt{x^2 + y^2} \qquad \qquad \qquad \textcolor{green}{m} := \frac{G}{c^2} \cdot m_{\text{zon}} \qquad \qquad m = 1.467 \times 10^3$$

$$c1(x,y) := \frac{\sqrt{1 - \frac{2 \cdot m}{\sqrt{x^2 + y^2}}}}{\sqrt{\left(1 - 2 \cdot \frac{m}{\sqrt{x^2 + y^2}}\right)^{-1}}}$$

$$\textcolor{red}{c}\left(0,690 \cdot 10^1\right) = \blacksquare$$

$$\sqrt{\frac{1 - \frac{2 \cdot m}{\sqrt{x^2 + y^2}}}{\left(1 - 2 \cdot \frac{m}{\sqrt{x^2 + y^2}}\right)^{-1}}}$$

$$\frac{2 \cdot m \cdot y \cdot \left(\frac{2 \cdot m}{\sqrt{x^2 + y^2}} - 1\right)}{\sqrt{\left(\frac{2 \cdot m}{\sqrt{x^2 + y^2}} - 1\right)^2 \cdot \left(x^2 + y^2\right)^{\frac{3}{2}}}}$$

$$dc dy(x,y) := - \frac{2 \cdot m \cdot y \cdot \left(\frac{2 \cdot m}{\sqrt{x^2 + y^2}} - 1\right)}{\sqrt{\left(\frac{2 \cdot m}{\sqrt{x^2 + y^2}} - 1\right)^2 \cdot \left(x^2 + y^2\right)^{\frac{3}{2}}}}$$

$$dc dy\Big(10,10^4\Big) = 2.934796 \times 10^{-5}$$

$$\text{hoek1}(\text{horleng}, y) := \int_{\frac{-\text{horleng}}{2}}^{\frac{\text{horleng}}{2}} \text{dcdy}(x, y) \, dx$$

$$a := \text{hoek1}\Big(680\cdot 10^8, 680\cdot 10^6\Big)$$

$$a = 8.636 \times 10^{-6}$$

$$\text{arcsec}(a) := a\cdot 206265 \qquad \text{arcsec}(a) = 1.781360369325$$

nauwkeuriger berekening

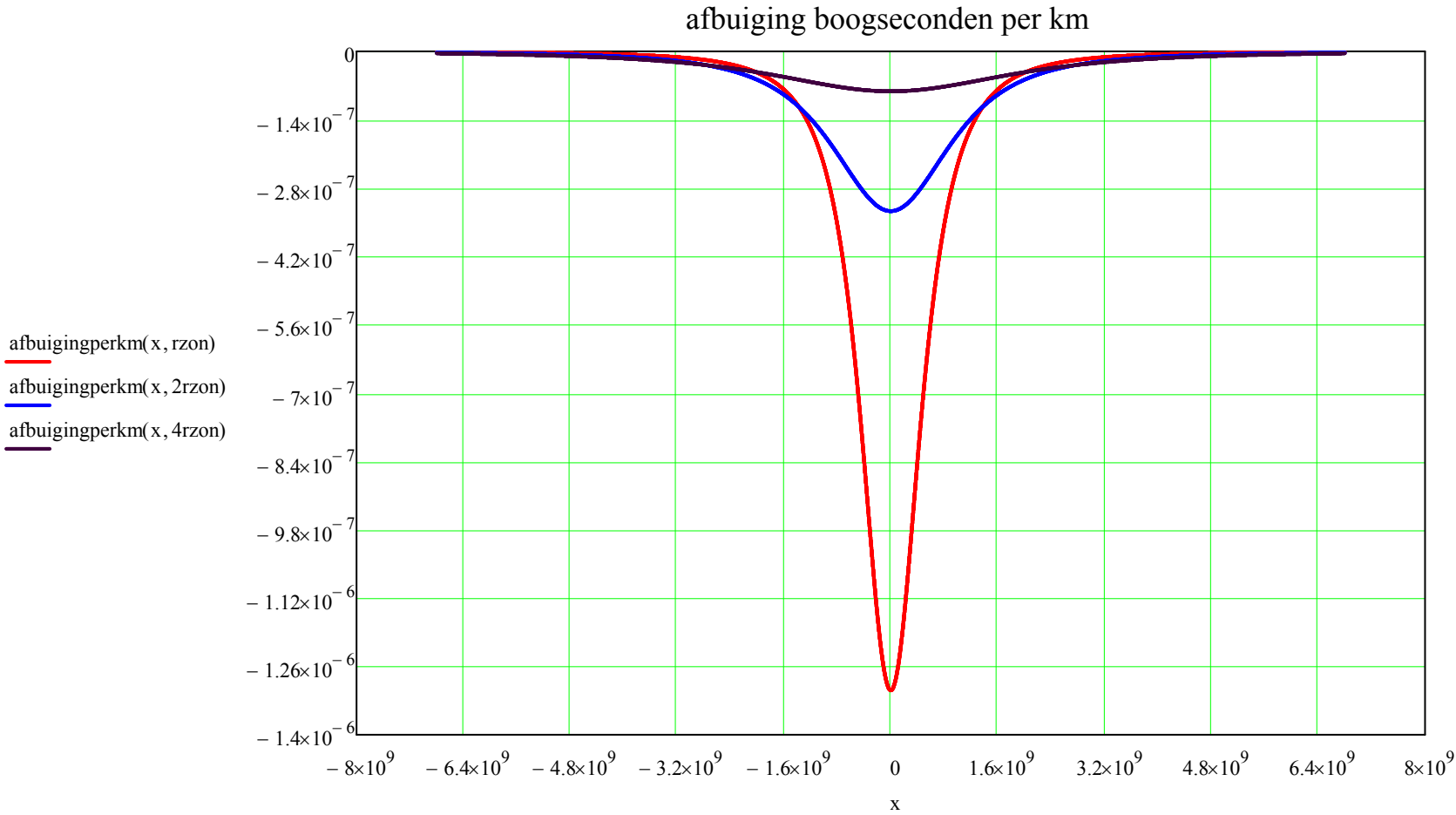
$$\begin{array}{l} \text{hoek}(\text{horleng}, y) := \left| \begin{array}{l} \text{deltax} \leftarrow 1000000 \\ \text{som} \leftarrow 0 \\ \text{boogseconden_rad} \leftarrow 206265 \\ x \leftarrow \frac{-\text{horleng}}{2} \\ \text{while } x < \frac{\text{horleng}}{2} \\ \quad \left| \begin{array}{l} \text{bijdrage} \leftarrow \text{dcdy}(x, y) \cdot \text{deltax} \cdot \text{boogseconden_rad} \\ \text{som} \leftarrow \text{som} + \text{bijdrage} \\ x \leftarrow x + \text{deltax} \end{array} \right. \\ \text{som} \end{array} \right. \end{array}$$

$$b := \text{hoek}\Big(680\cdot 10^7, 680\cdot 10^6\Big)$$

$$b = 1.745856$$

$$\text{afbuigingperkm}(x, y) := -\text{dcdy}(x, y) \cdot 1000 \cdot 206265$$

$$x := -10\cdot \text{rzon}, -10\cdot \text{rzon} + 100000..10\cdot \text{rzon}$$



$$c2(r) := \sqrt{1 - \frac{2\cdot m}{r}}$$

$$r = \sqrt{x^2 + y^2}$$

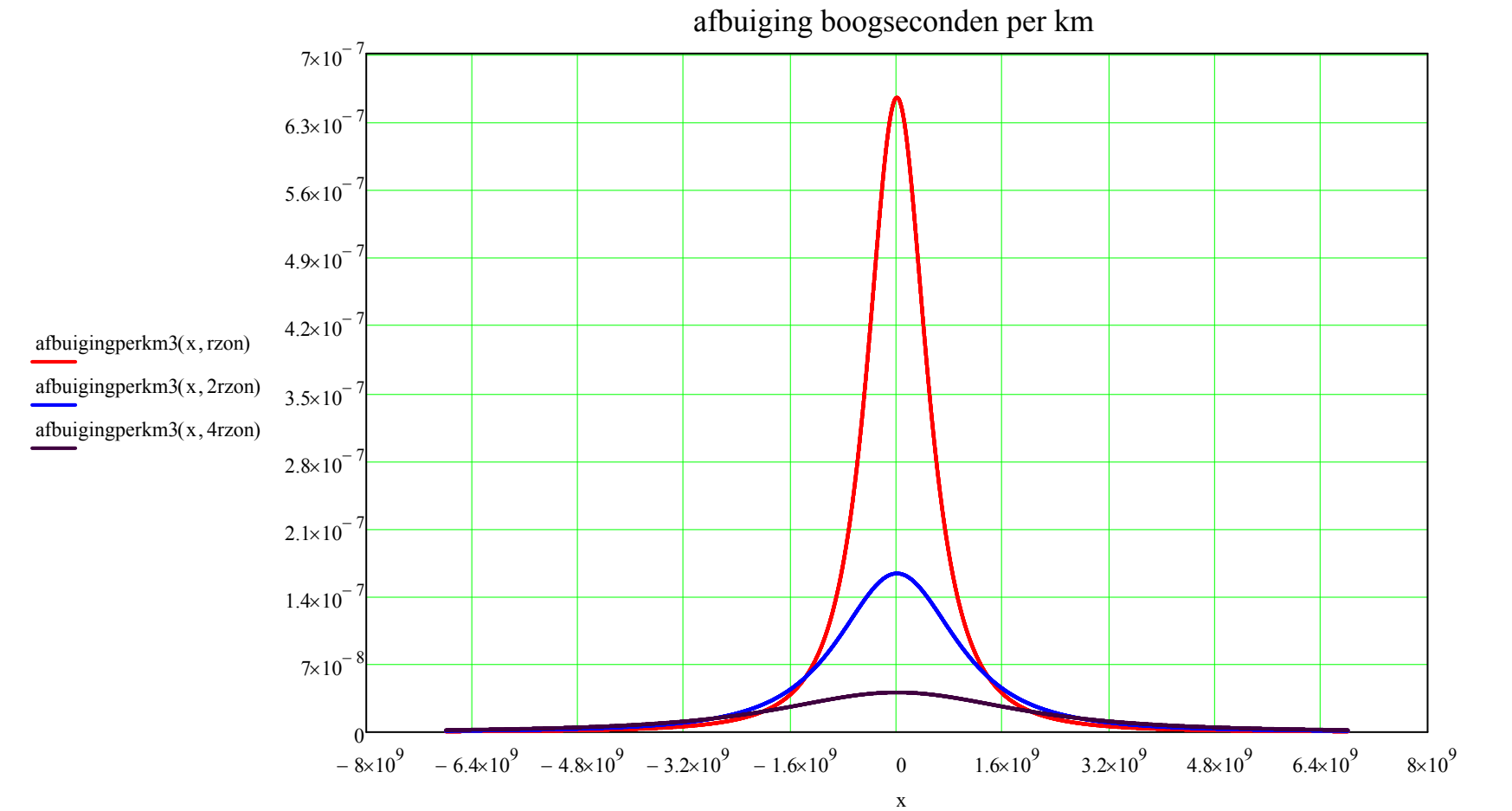
$$c3(x,y) := \sqrt{1 - \frac{2 \cdot m}{\sqrt{x^2 + y^2}}}$$

$$dcdy3(x,y) := \frac{m \cdot y}{\sqrt{1 - \frac{2 \cdot m}{\sqrt{x^2 + y^2}}} \cdot (x^2 + y^2)^{\frac{3}{2}}}$$

$$\text{hoek3}(\text{horleng},y) := \begin{array}{|l} \text{deltax} \leftarrow 1000000 \\ \text{som} \leftarrow 0 \\ \text{boogseconden_rad} \leftarrow 206265 \\ \text{x} \leftarrow \frac{-\text{horleng}}{2} \\ \text{while } \text{x} < \frac{\text{horleng}}{2} \\ \quad \begin{array}{|l} \text{bijdrage} \leftarrow \text{dcdy3}(\text{x},y) \cdot \text{deltax} \cdot \text{boogseconden_rad} \\ \text{som} \leftarrow \text{som} + \text{bijdrage} \\ \text{x} \leftarrow \text{x} + \text{deltax} \end{array} \\ \text{som} \end{array}$$

$$\text{afbuigingperkm3}(x,y) := \text{dcdy3}(x,y) \cdot 1000 \cdot 206265$$

$$x := -10 \cdot \text{rzon}, -10 \cdot \text{rzon} + 100000 .. 10 \cdot \text{rzon}$$



$$b3 := \text{hoek3}\Big(680 \cdot 10^7, 680 \cdot 10^6\Big)$$

$$b3 = 0.87293$$