https://www.mathpages.com/rr/s8-09/8-09.htm

As explained in Section 6.6, the line element for a spherically symmetrical gravitational field in the full theory of general relativity is given not by (1), but rather by

$$(d\tau)^2 = \left(1-\frac{2m}{r}\right)(dt)^2 - (dx)^2 - (dy)^2 - (dz)^2 \\ - \frac{1}{r^2}\left(\frac{2m}{r-2m}\right)(x\,dx + y\,dy + z\,dz)^2 \qquad (2)$$

This is just a re-writing of the Schwarzschild metric in quasi-Minkowskian coordinates, and it differs from (1) by the last term involving the space differentials. Since the "time-time" coefficient is the same, the frequency shift is unaffected, but we must account for the spatial curvature in our assessment of the directional deflection of light rays. Einstein announced this in the same paper of November 18, 1915, in which he presented for the first time the calculation of the anomalous precession of Mercury's orbit (see Sections 6.2 and 8.10). However, he merely summarized the new light deflection prediction in this paper, stating that

> This theory… produces an influence of the gravitational field on a light ray somewhat different from that given in my earlier work, because the velocity of light is determined by $g_{mn}dx^m dx^n = 0$. Upon the application of Huygens' principle, we find… after a simple calculation, that a light ray passing at a distance R suffers an angular deflection of magnitude 4m/R, while the earlier calculation… had produced the value 2m/R... In contrast to this difference, the result concerning the shift of the spectral lines by the gravitational potential… remains unaffected, because this result depends only on g_{tt}.

In his review article of 1916 he filled in the details of this "simple calculation", which is essentially just a repetition of the 1911 derivation, except that the variations of the spatial components of the metric are taken into account. In both derivations we have $d\tau = 0$, and we take $dy = dz = 0$ (to the first approximation) along the path, so the speed of light is

$$c(r) = \frac{dx}{dt} = \sqrt{-\frac{g_{tt}}{g_{xx}}}$$

The difference between the 1911 and 1915 derivations is simply that in 1911 Einstein took $g_{xx} = -1$, in accord with the quasi-Minkowskian metric (1), so he only considered the part of the deflection arising from the "time-time" component of the metric, whereas in November 1915 he realized that the full metric is actually given by (2), with varying spatial coefficients. In particular, we have

$$g_{xx} = -1 - \frac{x^2}{r^2}\left(\frac{2m}{r-2m}\right)$$

Therefore, the speed of light (along this path) is actually given, to the first order in the small quantities m/r, by the expression

$$c(r) = \sqrt{\frac{1 - \frac{2m}{r}}{1 + \frac{2m}{r}\frac{x^2}{r^2}\left(\frac{1}{1 - 2m/r}\right)}} \approx \left(1 - \frac{m}{r}\right)\left(1 - \frac{m}{r}\frac{x^2}{r^2}\right) \approx 1 - \frac{m}{r}\left(1 + \frac{x^2}{r^2}\right)$$

Now, proceeding just as he did in 1911, Einstein determined the total deflection using Huygens' principle by first evaluating the partial derivative

$$\frac{\partial c}{\partial y} = \frac{\partial}{\partial y}\left(1 - \frac{m}{\sqrt{x^2 + y^2}}\left(1 + \frac{x^2}{x^2 + y^2}\right)\right) = \frac{4x^2 + y^2}{r^5}\, ym$$