Define the operations $\circ$, $\square$ on the points of the euclidean plane as follows: $a \circ b$ is the point of trisection of the segment ab which lies nearer to a, whilst $a \square b$ is the midpoint of ab. Then (see diagram)

$$a \circ (b \square c) = x = (a \circ b) \square (a \circ c)$$

and

$$(b \square c) \circ a = y = (b \circ a) \square (c \circ a).$$

Thus $\circ$ is both left- and right-distributive over $\square$, but is not commutative, since clearly $a \circ b \neq b \circ a$ when a and b are distinct.

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60.21 Walk or run in the rain?

When faced with a short journey on foot in the rain, should one walk or run? Whilst my own inclination is usually to run, it does not seem obvious to me what action would minimise the total amount of rainwater to fall on me. A simple model and brief analysis of this problem is given here.

Assume that the rain is falling with a vertical component speed of q ft/sec, depositing water at the rate of g gallons/sq ft/sec on the ground. The wind, which we will assume for the moment is coming from the front, gives the rain a horizontal component speed of w ft/sec; that is, the apparent angle of the rain to a stationary man is $\tan^{-1}(w/q)$. I will model a person as a rectangular block of height h, shoulder length s and body thickness b, and assume that the rain will only fall upon his head and shoulders (area bs) and his front (area hs). He intends to travel a distance d and wishes to choose his speed v in a way which minimises the total amount of water W to hit him.

A moment's thought shows that the rain's contribution to the head and shoulders and to the front may be considered independently. The top is evidently collecting rain at a rate of $bs.g$ no matter at what speed the man is proceeding, giving a total of

$$bsg \, . \, \frac{d}{v} \cdot$$

The front is moving at an effective velocity of $v + w$ through water of density g/q gallons/cu ft, and hence collects it at a rate of $hs(v + w)g/q$, giving a total of

$$hs \, . \, \frac{g}{q} \, . \, (v + w) \, . \, \frac{d}{v} \cdot$$

The grand total, which may be written

$$W = sgd\left\{\frac{h}{q} + \frac{1}{v}\left(b + \frac{hw}{q}\right)\right\},$$

clearly decreases as v increases, and hence our man should move as quickly as possible, that is to say he should run.

Now suppose that the wind direction is from behind. There are two cases to consider, depending on whether the man's speed causes the rain to fall on his front or on his back, in other words $v > w$ or $v < w$.

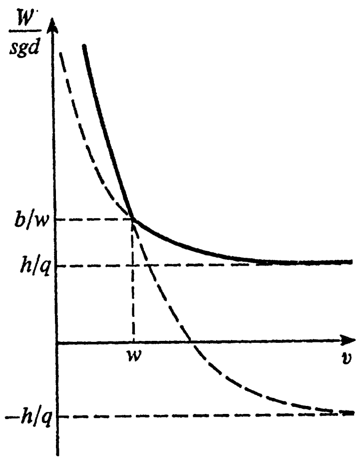
So long as $v < w$, then rain pours on his back at a rate $hs(w - v)g/q$, giving a grand total (back + top) of

$$W = \frac{hsg}{q}(w - v) \cdot \frac{d}{v} + bsg \cdot \frac{d}{v} = sgd\left\{-\frac{h}{q} + \frac{1}{v}\left(b + \frac{hw}{q}\right)\right\} \tag{1}$$

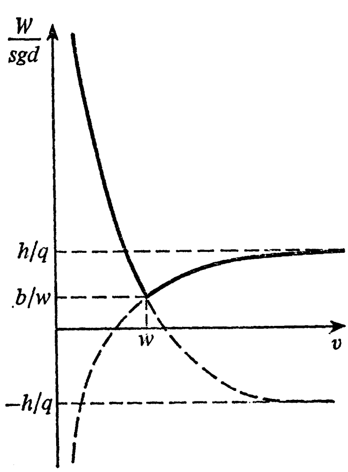
which is, once again, a function that decreases with v. Hence our man should increase his speed at least up to w. Beyond this, however, when $v > w$, the rain falls on his front at a rate $hs(v - w)g/q$, giving a grand total of

$$W = \frac{hsg}{q} \cdot (v - w) \cdot \frac{d}{v} + bsg \cdot \frac{d}{v} = sgd\left\{\frac{h}{q} + \frac{1}{v}\left(b - \frac{hw}{q}\right)\right\}. \tag{2}$$

If $b > hw/q$, then the function continues to decrease with v (Fig. 1(a)). We now come across the one situation in which running is not necessarily optimal. If $b < hw/q$ then the total rainwater W as a function of v has a



(a) The case $b > hw/q$.



(b) The case $b < hw/q$.

FIGURE 1

minimum at $v = w$, as shown in the Fig. 1(b). Note that the continuous line has equation (1) or (2) according as $v < w$ or $v > w$.

The condition $b < hw/q$ seems to be the most likely. A six foot man with a body thickness of one foot should move at velocity w so long as the rain falls at an angle of more than $9\frac{1}{2}°$ to the vertical from behind. Mind you, since rain falls at between 15 and 25 ft/sec, call it 22 ft/sec or 15 m.p.h., a 45° rain would have him running a four-minute mile to be optimal.

It could be that other factors, such as the presence of an umbrella, would alter the optimality criterion and so change the result, but otherwise the solution is to keep pace with the wind if it is from behind; otherwise, run for it.

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60.22 Are mudguards necessary?

Consider a wheel rolling on a road on which are particles of mud, and one particular particle of mass m which sticks to the wheel. Suppose the radial force required for dislodgement (apart from its weight, and disregarding any tendency to slide around the wheel) is kmg, where k is, perhaps, the coefficient of stickiness. Then when the radius makes an angle θ with the downward vertical, dislodgement will occur if kmg, together with the weight component, is insufficient to produce the required centripetal acceleration, i.e. if

$$kmg - mg\cos\theta < \frac{mv^2}{r},$$

where v is the vehicle's speed, or

$$\cos\theta > k - \frac{v^2}{gr}.$$

But $\cos\theta$ decreases from $+1$ to -1 as we pass from the lowest point of the wheel to the highest. So if v is sufficient for dislodgement it will occur at a very low value of θ or not at all. Therefore no particles of mud can leave the wheel elsewhere than very near the lowest point, whence they will be flung back on the road. Therefore mudguards are unnecessary.

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