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δ(r) :=
  δ1 ← 13.2 if (r ≥ 0) ∧ r < 400
  δ1 ← 13   if (r ≥ 400) ∧ r < 800
  δ1 ← 12.75 if (r ≥ 800) ∧ r < 1200
  δ1 ← 12    if (r ≥ 1200) ∧ r < 2200
  δ1 ← 11    if (r ≥ 2200) ∧ r < 2600
  δ1 ← 10.5  if (r ≥ 2600) ∧ r < 3000
  δ1 ← 10    if (r ≥ 3000) ∧ r < 3500
  δ1 ← 5.5   if (r ≥ 3500) ∧ r < 4000
  δ1 ← 5     if (r ≥ 4000) ∧ r < 4600
  δ1 ← 4.5   if (r ≥ 4600) ∧ r < 5200
  δ1 ← 4.5   if (r ≥ 4600) ∧ r < 5700
  δ1 ← 2     if (r ≥ 5700) ∧ r < 6500
  δ1

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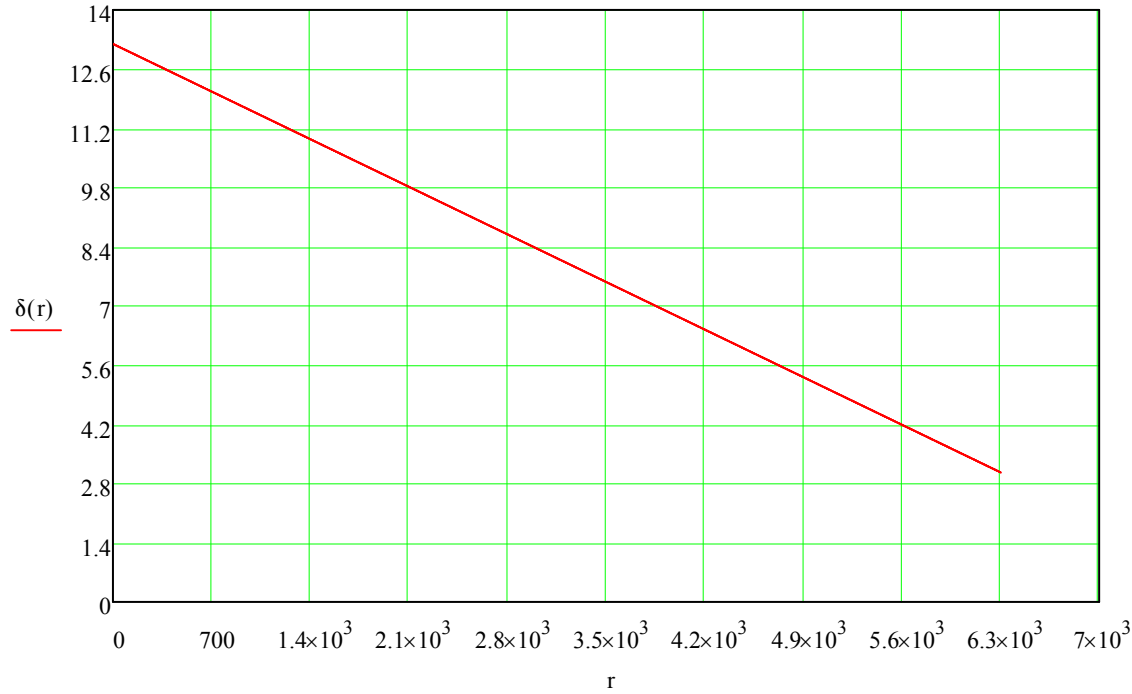
werkt nog niet goed

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δ(r) := 13.2 - (r / 6000) · 9.65
r := 0, 1..6300

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simpele benadering dichtheid



$$\frac{x^2+y^2}{a^2}+\frac{z^2}{b^2}=1$$

km:= 1000

b := 6356.7523km

G:= 6.6743·10<sup>-11</sup>

a := 6378.1370km

$\frac{a}{b}$  = 1.003

$$\frac{x^2+y^2}{a^2} = 1 - \frac{z^2}{b^2} \qquad x^2+y^2 = a^2 \cdot \left(1 - \frac{z^2}{b^2}\right) \qquad y = -\sqrt{a^2 \cdot \left(1 - \frac{z^2}{b^2}\right) - x^2}$$

$$y_n(x,z) := -\sqrt{a^2 \cdot \left(1 - \frac{z^2}{b^2}\right) - x^2} \qquad y_p(x,z) := \sqrt{a^2 \cdot \left(1 - \frac{z^2}{b^2}\right) - x^2}$$

$$x_n(y,z) := -\sqrt{a^2 \cdot \left(1 - \frac{z^2}{b^2}\right) - y^2} \qquad x_p(y,z) := \sqrt{a^2 \cdot \left(1 - \frac{z^2}{b^2}\right) - y^2}$$

$$r_1(x,y,z) := \sqrt{x^2+y^2+z^2}$$

$$z := b - 0.00001$$

$$\textcolor{green}{A}(z) := \int_{x_n(0,z)}^{x_p(0,z)} \int_{y_n(x,z)}^{y_p(x,z)} 1 \, dy \, dx$$

$$A(b - 0.001) = 4.021 \times 10^4 \quad \text{cirkel vlak bij de polen}$$

$$A(0) = 1.278 \times 10^{14} \quad \text{cirkel op evenaar}$$

$$\textcolor{green}{V} := \int_{-b}^b A(z) \, dz \qquad V = 1.083 \times 10^{12} \cdot \text{km}^3$$

$$V_1 := \int_{-b}^b \int_{x_n(0,z)}^{x_p(0,z)} \int_{y_n(x,z)}^{y_p(x,z)} 1 \, dy \, dx \, dz \qquad \text{alles opschrijven als 1 3dubbele integraal}$$

$$V_1 = 1.0832073174 \times 10^{12} \cdot \text{km}^3 \qquad \text{volume aarde}$$

$$\text{volume} = \int_{-b}^b \int_{x_n(0,z)}^{x_p(0,z)} \int_{y_n(x,z)}^{y_p(x,z)} r_1(x,y,z) \, dy \, dx \, dz$$

$$\text{volume} = \int_{-b}^b \int_{x_n(0,z)}^{x_p(0,z)} \int_{y_n(x,z)}^{y_p(x,z)} r_1(x,y,z) \, dy \, dx \, dz$$

$$M_1 := \int_{-b}^b \int_{x_n(0,z)}^{x_p(0,z)} \int_{y_n(x,z)}^{y_p(x,z)} \delta\left(\frac{r_1(x,y,z)}{\text{km}}\right) \cdot 1000 \, dy \, dx \, dz$$

$$M_1 = 5.974 \times 10^{24} \qquad \text{massa aarde} \qquad G = 6.674 \times 10^{-11}$$

$$\text{massa} = \int_{-b}^b \int_{x_n(0,z)}^{x_p(0,z)} \int_{y_n(x,z)}^{y_p(x,z)} \rho(r_1(x,y,z)) \, dy \, dx \, dz$$

$$p_1 := \begin{pmatrix} 0 \\ 6600\text{km} \\ 0 \end{pmatrix} \qquad p_2 := \begin{pmatrix} 0 \\ -6300\text{km} \\ 0 \end{pmatrix}$$

$$\text{lengte}(p_1,p_2) := \left| \begin{array}{l} \text{dist2} \leftarrow \left(p_{1_0} - p_{2_0}\right)^2 + \left(p_{1_1} - p_{2_1}\right)^2 + \left(p_{1_2} - p_{2_2}\right)^2 \\ \sqrt{\text{dist2}} \end{array} \right.$$

$$\text{lengte}(p_1,p_2) = \sqrt{\left(p_{1_0} - p_{2_0}\right)^2 + \left(p_{1_1} - p_{2_1}\right)^2 + \left(p_{1_2} - p_{2_2}\right)^2}$$

$$\text{lengte}(p_1,p_2) = 1.29 \times 10^7$$

$$F(p1,p2x,p2y,p2z,dm) := \left| \begin{array}{l} G \leftarrow 6.6743 \cdot 10^{-11} \\ p2_0 \leftarrow p2x \\ p2_1 \leftarrow p2y \\ p2_2 \leftarrow p2z \\ dist2 \leftarrow \left(p1_0 - p2_0\right)^2 + \left(p1_1 - p2_1\right)^2 + \left(p1_2 - p2_2\right)^2 \\ F1 \leftarrow \frac{G \cdot dm}{dist2} \\ r \leftarrow \frac{\begin{pmatrix} p1_0 - p2_0 \\ p1_1 - p2_1 \\ p1_2 - p2_2 \end{pmatrix}}{\sqrt{dist2}} \\ r \cdot F1 \end{array} \right.$$

$$F\Big(p1,p2_0,p2_1,p2_2,5.974\times 10^{24}\Big)=\begin{pmatrix} 0 \\ 2.396 \\ 0 \end{pmatrix}$$

$$g1x(p1) := \int_{-b}^b \int_{xn(0,z)}^{xp(0,z)} \int_{yn(x,z)}^{yp(x,z)} F\Big(p1,x,y,z,\delta\Big(\frac{r1(x,y,z)}{km}\Big)\cdot 1000\Big)_0 \, dy \, dx \, dz$$

$$g1(p1) := \left| \begin{array}{l} gx \leftarrow \int_{-b}^b \int_{xn(0,z)}^{xp(0,z)} \int_{yn(x,z)}^{yp(x,z)} F\Big(p1,x,y,z,\delta\Big(\frac{r1(x,y,z)}{km}\Big)\cdot 1000\Big)_0 \, dy \, dx \, dz \\ gy \leftarrow \int_{-b}^b \int_{xn(0,z)}^{xp(0,z)} \int_{yn(x,z)}^{yp(x,z)} F\Big(p1,x,y,z,\delta\Big(\frac{r1(x,y,z)}{km}\Big)\cdot 1000\Big)_1 \, dy \, dx \, dz \\ gz \leftarrow \int_{-b}^b \int_{xn(0,z)}^{xp(0,z)} \int_{yn(x,z)}^{yp(x,z)} F\Big(p1,x,y,z,\delta\Big(\frac{r1(x,y,z)}{km}\Big)\cdot 1000\Big)_2 \, dy \, dx \, dz \\ g_0 \leftarrow gx \\ g_1 \leftarrow gy \\ g_2 \leftarrow gz \\ g \end{array} \right.$$

$$p1 = \begin{pmatrix} 0 \\ 6.6 \times 10^3 \\ 0 \end{pmatrix} \cdot km \qquad p1a := \begin{pmatrix} a + 1km \\ 0 \\ 0 \end{pmatrix} \qquad p1b := \begin{pmatrix} 0 \\ 0 \\ b + 1km \end{pmatrix} \qquad nul := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$g1(p1) = \begin{pmatrix} 0 \\ 9.162 \\ 0 \end{pmatrix} \qquad g1(p1a) = \begin{pmatrix} 9.809 \\ 0 \\ 0 \end{pmatrix} \qquad g1(p1b) = \begin{pmatrix} 0 \\ 0 \\ 9.843 \end{pmatrix}$$

$$lengte(nul,p1a) = 6.379 \times 10^6$$

$$lengte(nul,g1(p1a)) = 9.809$$

```

g_angle(a,b,α,h) :=
  y ← 0
  k ← 1.01
  z ← b·cos(α)
  x ← √{a2·(1 - z2/b2) - y2}
  p10 ← x + h·sin(α)
  p11 ← 0
  p12 ← z + h·cos(α)
  g ← g1(p1·k)·k2
  g_angle ← lengte(g,nul)
  g_angle

```

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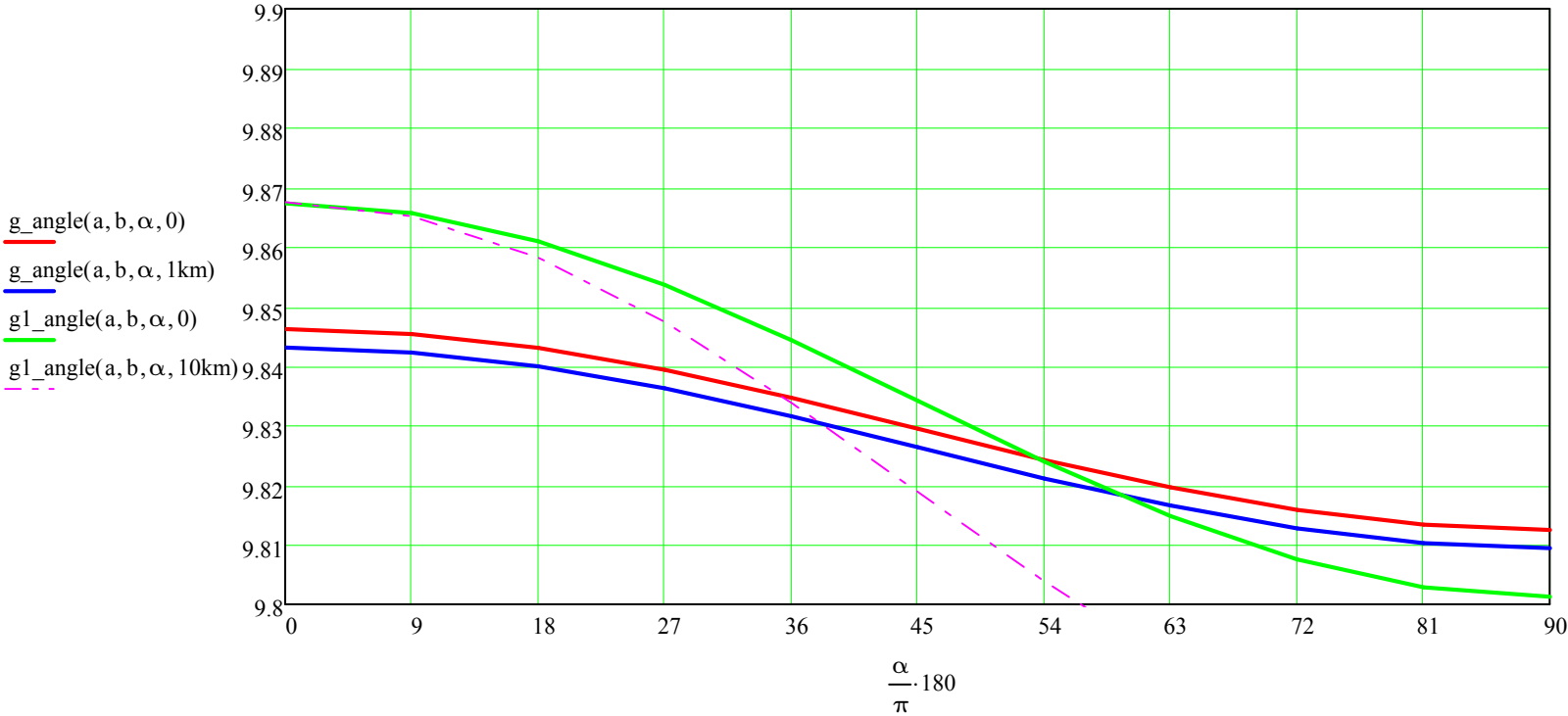
gl_angle(a,b,α,h) :=
  y ← 0
  Ma ← 5.974·1024
  z ← b·cos(α) + h·cos(α)
  x ← √{a2·(1 - z2/b2) - y2} + h·sin(α)
  g ← G·Ma / (x2 + y2 + z2)
  g

```

$$\alpha := 0, \frac{\pi}{20} .. \frac{\pi}{2}$$

$$k1 := 1.005$$

$$k2 := 1.01$$



$$\frac{g\_angle(a,b,0,0)}{g\_angle\left(a,b,\frac{\pi}{2},0\right)} = 1.00344$$

ratio of gravity at pole and equator with actual mass distribution

$$\frac{gl\_angle(a,b,0,0)}{gl\_angle\left(a,b,\frac{\pi}{2},0\right)} = 1.00674$$

ratio of gravity at pole and equator with mass at centre of earth

```

gcentripet(a,b,α,h) :=
  z ← b·cos(α)
  x ← √{a2·(1 - z2/b2)}
  r2 ← x2 + z2
  r ← √r2 + h
  r_rot ← r·sin(α)
  v ← 2·π·r_rot / 24·3600
  gcentripet ← v2 / (r_rot + 0.01)
  gcentripet

```

$$gcentripet(a,b,90,0) = 0.030135$$

$$a \cdot 2 \cdot \frac{\pi}{24} = 1.67 \times 10^3 \text{ km}$$

$$b = 6.357 \times 10^3 \text{ km}$$

$$\text{gtotaal}(a,b,\text{breedtegraad},\text{hoogte}) := \left| \begin{array}{l} \alpha \leftarrow \frac{\pi}{2} - \frac{\text{breedtegraad}}{180} \cdot \pi \\ \text{gzw} \leftarrow \text{g\_angle}(a,b,\alpha,\text{hoogte}) \\ \text{gzw1} \leftarrow \text{gl\_angle}(a,b,\alpha,\text{hoogte}) \\ \text{gcentr} \leftarrow \text{gcentripet}(a,b,\alpha,\text{hoogte}) \\ \text{gtotaal}_0 \leftarrow \text{gzw} \\ \text{gtotaal}_1 \leftarrow \text{gzw1} \\ \text{gtotaal}_2 \leftarrow \text{gcentr} \\ \text{gtotaal}_3 \leftarrow \text{gzw} - \text{gcentr} \\ \text{gtotaal} \end{array} \right.$$

$$\text{amsterdam} := 52.351 \qquad \text{chimay} := 50.048 \qquad \text{pool} := 90 \qquad \text{evenaar} := 0 \qquad \text{h\_amsterdam} := 0 \qquad \text{h\_chimay} := 236$$

$$\text{gtotaal}(a,b,\text{amsterdam},0) = \begin{pmatrix} 9.834 \\ 9.843 \\ 0.021 \\ 9.813 \end{pmatrix} \qquad \text{gtotaal}(a,b,\text{chimay},0) = \begin{pmatrix} 9.832 \\ 9.84 \\ 0.022 \\ 9.811 \end{pmatrix}$$

$$\text{gewicht}(\text{breedtegraad},\text{refbreedtegraad},\text{refgewicht},\text{refhoogte},\text{hoogte}) := \text{refgewicht} \cdot \frac{\text{gtotaal}(a,b,\text{breedtegraad},\text{hoogte})_3}{\text{gtotaal}(a,b,\text{refbreedtegraad},\text{refhoogte})_3}$$

$$\text{gewicht}(\text{amsterdam},\text{amsterdam},500,\text{h\_amsterdam},\text{h\_amsterdam}) = 500$$

$$\text{gewicht}(\text{chimay},\text{amsterdam},500,\text{h\_amsterdam},\text{h\_chimay}) = 499.844$$

$$\text{gewicht}(\text{evenaar},\text{amsterdam},500,\text{h\_amsterdam},0) = 498.248$$

$$\text{gewicht}(\text{pool},\text{amsterdam},500,\text{h\_amsterdam},0) = 501.685$$

$$\text{gewicht}(\text{evenaar},\text{amsterdam},500,\text{h\_amsterdam},42246\text{km} - a) = -2.328 \times 10^{-4}$$

$$\text{gewicht}(\text{amsterdam},\text{amsterdam},500,\text{h\_amsterdam},\text{h\_amsterdam}) - \text{gewicht}(\text{chimay},\text{amsterdam},500,\text{h\_amsterdam},\text{h\_chimay}) = 0.156$$